Energy Detection—Based Spectrum Sensing over $\kappa-\mu$ and $\kappa-\mu$ Extreme Fading Channels

P. C. Sofotasios, Member, IEEE, E. Rebeiz, Student Member, IEEE, Li Zhang, Senior Member, IEEE, T. A. Tsiftsis, Senior Member, IEEE, S. Freear, Senior Member, IEEE, and D. Cabric, Member, IEEE

Abstract—Energy detection is a simple and popular method of spectrum sensing in cognitive radio systems. It is also widely known that the performance of sensing techniques is largely affected when users experience fading effects. This paper investigates the performance of an energy detector over the generalized $\kappa-\mu$ and $\kappa-\mu$ Extreme fading channels which have been shown to provide remarkably accurate fading characterization. Novel analytic expressions are firstly derived for the corresponding average probability of detection for the case of single user detection. These results are subsequently extended to the case of square-law selection diversity as well as for collaborative detection scenarios. As expected, the performance of the detector is highly dependent upon the severity of fading since even small variation of the fading conditions affect significantly the value of the average probability of detection. Furthermore, the performance of the detector improves substantially as the number of branches or collaborating users increase in both severe and moderate fading conditions while it is shown that the $\kappa-\mu$ Extreme model is capable of accounting for fading variations even at low signal-to-noise values. The offered results are particularly useful in assessing the effect of fading in energy detection-based cognitive radio communication systems and therefore they can be used in quantifying the associated trade-offs between sensing performance and energy efficiency in cognitive radio networks.

Index Terms—Spectrum sensing, energy detector, fading channels, unknown signal detection, collaborative spectrum sensing, $\kappa-\mu$ fading, diversity.

I. INTRODUCTION

The detection of unknown signals is an important topic in wireless communications. It is typically realized in the form of spectrum sensing with energy detection (ED) constituting the most simple and popular method. The operating principle of ED is based on the deployment of a radiometer, which is a non-coherent detection device that measures the energy level of a received signal waveform over an observation time window. Then, it compares it with a pre-defined energy threshold and determines accordingly whether an unknown signal is present or absent [1]–[3].

H. Urkowitz was the first to address the problem of detecting unknown signals over a flat band-limited Gaussian noise channel and comprehensively derived analytic expressions for the probability of detection and the probability of false alarm [4]. These performance metrics are based on the assumption that the decision statistics follow the central chi-square and the non-central chi-square distribution, respectively. A few decades later, this problem was revisited by Kostylev who considered quasi-deterministic signals operating in fading environments [5]. Thanks to its low implementation complexity and no requirements for knowledge of the signal, the ED method has been widely associated with applications in RADAR systems while it has been also shown to be rather applicable in emerging wireless technologies such as ultra-wideband communications and cognitive radio [6], [7]. In the former, the energy detector is exploited for borrowing an idle channel from authorized users, whereas in the latter it identifies the presence or absence of a deterministic signal and decides whether a primary licensed user is active or idle, respectively. According to the corresponding decision, the secondary unlicensed user either remains silent, or proceeds in utilizing the unoccupied band until the state of the primary user becomes active [1], [8]. This opportunistic method has been extensively shown to substantially increase the utilization of the already allocated radio spectrum, offering a considerable mitigation of the currently extensive spectrum scarcity [9].

Capitalizing on the above, numerous studies have been devoted to the analysis of the performance of energy detection-based spectrum sensing for different communication scenarios. Specifically, the authors in [10] derived closed-form expressions for the average probability of detection over Rayleigh, Rice and Nakagami fading channels for both single-channel and multi-channel scenarios. Likewise, the ED performance in the case of equal gain combining and Nakagami-$m$ multipath fading has been investigated in [11] whereas the corresponding performance in collaborative spectrum sensing and in relay-based cognitive radio networks has been evaluated in [12]–[16]. A novel semi-analytic method for analyzing the performance of energy detection of unknown deterministic signals was reported in [17]. This important work is based on the moment-generating function (MGF) method and aims to overcome the analytical difficulties that arise from the presence of the Marcum Q-function. This method was utilized in the case of maximum-ratio combining (MRC) in the presence of Rayleigh, Rice and Nakagami-$m$ fading in [17] as well as for the useful case of correlated Rayleigh and Rician fading channels in [18]. Finally, the detection of unknown signals
in low signal-to-noise-ratio (SNR) and in K-distributed \((K)\),
generalized \(K (K_G)\) and the very flexible \(\eta-\mu\) fading channels
has been recently analyzed in [19]–[22].

The \(\kappa-\mu\) distribution is a generalized fading model that is
distinctive for providing adequate characterization of multipath
fading particularly for line-of-sight (LOS) communication
scenarios. It was reported in [23] along with the \(\eta-\mu\) fading
model which accounts for non-line-of-sight (NLOS) communica-
tion conditions. The \(\kappa-\mu\) fading model has been shown to
be particularly flexible and it includes as special cases the well
known Rice, Nakagami-\(\eta\), Rayleigh and one-sided Gaussian
distributions [23]. Its remarkable flexibility and usefulness are
shown clearly in [23, Fig. 9]. The known \(\eta-\mu\) fading model is also
depicted and one can notice how the two models complement each-other. It also evident from this Figure that the \(\kappa-\mu\) fading model significantly outperforms the characterization
capabilities of Rice, Nakagami-\(\eta\) and Rayleigh distributions.
Furthermore, it constitutes the basis for deriving the \(\kappa-\mu\) extreme
distribution which is a recently proposed remarkable fading
model that provides accurate characterization of radio propagation under severe fading conditions. Importantly, it has been
shown that these conditions, even worse than Rayleigh, typically occur in enclosed environments such as airplanes,
trains, buses and shopping malls [24], [25].

Nevertheless, in spite of the undoubted usefulness of the
\(\kappa-\mu\) and \(\kappa-\mu\) extreme fading models, no studies related to
the detection of unknown signals over these fading channels have been reported in the open technical literature. Motivated
by this, this work is devoted to the analysis of energy detection
over \(\kappa-\mu\) and \(\kappa-\mu\) extreme fading channels. Specifically,
novel analytic expressions are firstly derived for the average
probability of detection for the conventional single user scen-
ario. Subsequently, these expressions are extended to account for
the collaborative energy detection as well as for the square-law-selec-
tion (SLS) diversity scheme. As expected, the performance of the energy detector is highly dependent upon
the value and variations of the \(\kappa\) and \(\mu\) fading parameters.
This is evident in the whole range of the average SNR and
particularly in the positive regime, as even small variations of
the fading conditions affect significantly the performance of the corresponding average probability of detection. In addition,
it is shown that the \(\kappa-\mu\) extreme model appears capable of
accounting for fading variations even at low SNR values.
Finally, it is shown that the detector’s performance is, as
expected, substantially improved in both severe and moderate
fading conditions as the number of diversity paths or number of
users increase. Therefore, the offered results enable us
to quantify the effect of fading in the system performance in
various communication scenarios and thus to determine
the required power levels for ensuring robust and accurate
performance of energy detectors combined with sufficient
benefits in terms of energy efficiency.

The remainder of this paper is organized as follows: The
system and channel model are described in Section II. The
average detection probabilities of the energy detector over
\(\kappa-\mu\) and \(\kappa-\mu\) extreme channels are analyzed in Sections III
and IV, respectively. Numerical results for each communica-
tion scenario and discussions are provided in Section V, while

II. SYSTEM AND CHANNEL MODEL

A. Energy Detection

In narrowband energy detection, the received signal wave-
form follows a binary hypothesis that can be represented
according to [18, eq. (1)],

\[
r(t) = \begin{cases} 
  n(t) & : H_0 \\
  h s(t) + n(t) & : H_1 
\end{cases} 
\]

where \(s(t)\) is an unknown deterministic signal whereas \(h\)
denotes the amplitude of the channel coefficient and \(n(t)\) is an
additive white Gaussian noise (AWGN) process. The samples of
\(n(t)\) are assumed to be zero-mean Gaussian random vari-
ables with variance \(N_0 W\) with \(W\) and \(N_0\) denoting the single-
sided signal bandwidth and a single-sided noise power spectral
density, respectively [18]. The hypotheses \(H_0\) and \(H_1\) refer to
the cases that a signal is absent or present, respectively. The
received signal is subject to filtering, squaring and integration over
the time interval \(T\) which is expressed as [10, eq. (2)],

\[
y \triangleq \frac{2}{N_0} \int_0^T |r(t)|^2 \, dt.
\]

The output of the integrator is a measure of the energy of
the received waveform which acts as a test statistic that
determines whether the received energy measure corresponds
only to the energy of noise \((H_0)\) or to the energy of both the
unknown deterministic signal and noise \((H_1)\). By denoting the
time bandwidth product as \(u = TW\), the test statistic follows
the central chi-square distribution with \(2u\) degrees of
freedom under the \(H_0\) hypothesis and the non central chi-
square distribution with \(2u\) degrees of freedom under the \(H_1\)
hypothesis [4]. To this effect, the corresponding probability
density function (PDF) in the presence of AWGN is given by
[10, eq. (3)], namely,

\[
p_y(y) = \begin{cases} 
  \frac{1}{2\Gamma(u)} y^{u-1} e^{-\frac{y}{2}} & : H_0 \\
  \frac{1}{2} \left( \frac{y}{y^*} \right)^{u-1} e^{-\frac{y+y^*}{2}} I_{u-1} \left( \sqrt{2y\gamma} \right) & : H_1 
\end{cases}
\]

where \(\gamma \triangleq \frac{1}{4} h^2 E_s/N_0\) is the SNR and \(E_s, T\) denote
the signal energy and the observation time interval, respectively.
Also, \(\Gamma(a) \triangleq \int_0^\infty t^{a-1} e^{-t} \, dt\) is the Euler’s gamma function
while \(I_n(x) \triangleq \frac{1}{\pi} \int_0^\infty \cos(n\theta)e^{-x\cos(\theta)} \, d\theta\) is the modified Bessel
function of the first kind [26].

As already mentioned, an energy detector is largely charac-
terized by a predefined energy threshold, \(\lambda\). This threshold
is critical in the decision process and is promptly associated to
three measures that overall evaluate the performance of the
detector: i) the probability of false alarm, \(P_f \triangleq Pr(y > \lambda \mid H_0)\);
ii) the probability of detection, \(P_d \triangleq Pr(y > \lambda \mid H_1)\) and iii) the
probability of missed detection, \(P_m = 1 - P_d\). The first two
measures are deduced by integrating (2) over zero to infinity
yielding [10],

\[
P_f = \frac{\Gamma(u, \frac{\lambda}{2})}{\Gamma(u)} \tag{3}
\]

and

\[
P_d = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) \tag{4}
\]
where \( \Gamma(a, x) \triangleq \int_x^\infty t^{a-1}e^{-t}dt \) and \( Q_m(a, b) \triangleq \frac{1}{\alpha_m} \int_b^\infty \frac{x^{\alpha+m-2}e^{-\frac{x^2}{2}}}{I_{m-1}(ax)}dx \) denote the upper incomplete gamma function and the generalized Marcum Q-function, respectively [26, 27].

B. The \( \kappa-\mu \) Fading Model

As already mentioned in Sec. I, the \( \kappa-\mu \) fading model has been shown to represent effectively the small-scale variations of a fading signal in LOS communications. Physically, this fading model considers a signal composed of clusters of multipath waves propagating in a non-homogeneous environment. Within any one cluster, the phases of the scattered waves are random and have similar delay times with delay-time spreads of different clusters being relatively large. The clusters of multipath waves are assumed to have scattered waves with identical powers while each cluster consists of a dominant component with arbitrary power. To this effect, the parameters \( \mu \) and \( \kappa \) correspond to the number of multipath clusters and the ratio between the total power of the dominant components and the total power of the scattered waves, respectively. These two parameters render this fading model remarkably flexible as its capturing range is particularly broad [23, Fig. 9]. This is also evident by the fact that the widely known Rice and Nakagami-\( m \) fading models are included as special cases for \( \mu = 1 \) and \( \kappa = 0 \), respectively [23]. Therefore, this model can provide a meaningful insight on how fading affects the performance of an energy detector which ultimately leads to a significant improvement on the design of cognitive radio systems in terms of energy efficiency and cost.

For a fading signal with envelope \( P = R/\hat{r} \) and \( \hat{r} = \sqrt{E(R^2)} \), the envelope PDF of \( \kappa-\mu \) distribution is expressed as follows [23, eq. (1)],

\[
p_{\hat{r}}(r) = \frac{2\mu(\kappa + 1)\mu^\kappa e^{-\mu\hat{r}} \hat{r}^\mu}{\Gamma(\kappa + 1)\hat{r}^{\mu + 1}} I_{\mu-1} \left( 2\mu \sqrt{\kappa(\kappa + 1)\gamma} \right)
\]

(5)

where \( E(.) \) denotes statistical expectation and \( \hat{r} \) is the root-mean-square (RMS) value of \( R \). The corresponding PDF of the instantaneous SNR per symbol \( \gamma \) is given by [23, eq. (10)],

\[
p_{\gamma}(\gamma) = \frac{\mu(\kappa + 1)\mu^\kappa e^{-\mu\gamma} \gamma^{\mu+1}}{\Gamma(\kappa + 1)\gamma^{\mu + 1}} I_{\mu-1} \left( 2\mu \sqrt{\kappa(\kappa + 1)\gamma} \right)
\]

(6)

where \( \gamma \) represents the average SNR per symbol. Furthermore, the fading parameters \( \kappa \) and \( \mu \) are related to each other by the following relationship [23],

\[
\mu = \frac{E^2(R^2)}{\text{Var}(R^2)} \frac{1 + 2\kappa}{(1 + \kappa)^2} = \frac{E^2(P)}{\text{Var}(P)} \frac{1 + 2\kappa}{(1 + \kappa)^2}
\]

(7)

with \( \text{Var}(.) \) denoting mathematical variance.

III. AVERAGE DETECTION PROBABILITY OVER \( \kappa-\mu \) FADING CHANNELS

It is recalled that (3) and (4) account for the case of AWGN channels. For communication scenarios over fading channels the average probability of detection is obtained by averaging (4) over the corresponding SNR fading statistics, namely,

\[
P_d = \int_0^\infty Q_u(\sqrt{2\gamma}, \sqrt{\lambda})p_{\gamma}(\gamma)\,d\gamma.
\]

(8)

Based on this, analytic expressions for the case of Rayleigh, Rice, Nakagami-\( m \), \( K \), \( K_G \), log-normal and \( \eta-\mu \) fading channels where derived in [8]–[14], [17]–[22]. Therefore, the average probability of detection in the case of generalized \( \kappa-\mu \) fading can be obtained by averaging (4) over the statistics of (6). To this effect, by substituting (6) in (8) yields,

\[
P_{d_{\kappa-\mu}} = \int_0^\infty \mu Q_u \left( \sqrt{2\gamma}, \sqrt{\lambda} \right) I_{\mu-1} \left( 2\mu \sqrt{\kappa(\kappa + 1)\gamma} \right) d\gamma.
\]

(9)

An analytic expression for (9) can be derived by expressing the involved Marcum Q-function according to [28, eq. (29)] and [29, eq. (6)], as follows:

\[
Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) = e^{-\gamma} \sum_{l=0}^\infty \frac{\gamma^l \Gamma(l + u, \frac{\lambda}{\gamma})}{\Gamma(l + 1)\Gamma(l + u)}
\]

(10)

Hence, by substituting (10) into (9), \( P_{d_{\kappa-\mu}} \) is re-written as,

\[
P_{d_{\kappa-\mu}} = \frac{\mu(\kappa + 1)\mu^\kappa e^{-\mu\gamma} \gamma^{\mu+1}}{\Gamma(\kappa + 1)\gamma^{\mu + 1}} \sum_{l=0}^\infty \frac{\Gamma(l + u, \frac{\lambda}{\gamma})}{\Gamma(l + 1)\Gamma(l + u)}
\]

(11)

\[
\times \int_0^\gamma I_{\mu-1} \left( 2\mu \sqrt{\kappa(\kappa + 1)\gamma} \right) d\gamma.
\]

Notably, the above integral is identical to [30, eq. (6.643.2)] which can be expressed in terms of the Whittaker hypergeometric function, \( M_{\nu, \mu}(z) \) given by,

\[
\mathcal{I}_1 = (\mu) e^{\frac{\nu(\kappa+\lambda+2z)}{2}} M_{-\frac{1}{2} - \frac{\lambda}{2} - \frac{\nu}{2}} \left( \frac{\kappa(1+\lambda)\mu^2}{\nu + \mu(1+\kappa)} \right)
\]

(12)

where \( (\mu) \triangleq \Gamma(\mu + l)/\Gamma(\mu) \) is the Pochhammer symbol [26]. With the aid of [30, eq. (9.220.2)], equation (12) can be equivalently expressed as

\[
\mathcal{I}_1 = (\mu)^l \mu^\kappa \kappa^l (1 + \kappa + \lambda) F_1 \left( \mu + l; \mu; \frac{\kappa(1+\lambda)\mu^2}{\nu + \mu(1+\kappa)} \right)
\]

(13)

where \( F_1(a; b; x) \) denotes the Kummer’s confluent hypergeometric function [26]. Hence, by substituting (13) into (11), the following infinite series representation for \( P_{d_{\kappa-\mu}} \) is deduced:

\[
P_{d_{\kappa-\mu}} = \sum_{l=0}^\infty \frac{\mu^l(\kappa+\lambda)e^{-\mu\gamma}(l + u)\gamma^{l+\lambda}}{\Gamma(l + 1)\Gamma(\mu + l + \nu)\Gamma(l + u, \frac{\lambda}{\gamma})}
\]

(14)

with

\[
F_A \left( \mu; 1; 1, \frac{\nu}{\nu + \mu\kappa}; \kappa(1+\lambda)\mu^2 \right) - \frac{H_A \left( \mu, u, u + 1, \mu; \frac{\lambda}{\nu + \mu\kappa}; \kappa(1+\lambda)\mu^2 \right) \nu^{2\lambda} u^{\lambda-1}}{n!}
\]

(15)

\[n!\equiv1\text{ for } k\to\infty\]

One can alternatively use the accurate polynomial approximation in [29, eq. (7)], [41, eq. (3.39)] which reduces to [29, eq. (7)] for \( k \to \infty \).
where $F_A()$ and $H_A()$ denote the Lauricella hypergeometric function of two variables and the complete triple hypergeometric function of the first kind, respectively [30], [33]–[36]. A detailed proof of (15) is provided in the Appendix.

A. Square-Law Selection (SLS)

Square-Law selection, also known as selection combining, is an efficient diversity scheme that is highly regarded for its simple realization. Its principle of operation is based on selecting the output of the branch with maximum decision statistic, $b_{SLS} = \max \{y_1, y_2, \ldots, y_L\}$, [31]. Based on this, the corresponding $P_d$ in the case of $\kappa-\mu$ fading can be obtained by averaging $P_d^{SLS}$ in [10, eq. (15)] over $L$ independent $\kappa-\mu$ branches, namely,

$$P_d^{SLS} = 1 - \prod_{i=1}^{L} \int_{0}^{\infty} [1 - Q_u \left( \sqrt{2}\gamma_i, \sqrt{\lambda} \right)] p_{\gamma_i}(\gamma_i) d\gamma_i.$$  

Therefore, by substituting (6) in (16) it follows that

$$P_d^{SLS} = 1 - \prod_{i=1}^{L} \left[ \int_{0}^{\infty} p_{\gamma_i}(\gamma_i) d\gamma_i - \int_{0}^{\infty} Q_u \left( \sqrt{2}\gamma_i, \sqrt{\lambda} \right) p_{\gamma_i}(\gamma_i) d\gamma_i \right].$$  

By recalling that $\int_{0}^{\infty} p(\gamma) d\gamma = 1$, it is noticed that the integral that needs to be evaluated in (17) is the same as the integral in (9). Therefore, by following the same procedure as in the derivation of $P_d^{SLS}$, a closed-form expression for (17) is deduced, namely,

$$P_d^{SLS} = 1 - \prod_{i=1}^{L} \left\{ \left[ 1 - P_d^{SLS}(\gamma_i) \right] \right\}$$

$$= 1 - \prod_{i=1}^{L} \left\{ 1 - \frac{\mu^\mu (1 + \kappa)^\mu e^{-\mu\kappa \gamma} [\frac{\kappa (1 + \kappa)^{2}}{\gamma \mu + \mu (1 + \kappa)}]}{\Gamma(u) \gamma^u e^{-\gamma} \frac{\Gamma(1 + \kappa)}{(\gamma + \mu (1 + \kappa))^{1 + \kappa}}} \right\}.$$  

As always, the corresponding $P_f$ is independent of the fading statistics and is given by [10, eq. (14)],

$$P_f^{SLS} = 1 - \left[ 1 - \frac{\Gamma(u, \frac{\lambda}{\gamma})}{\Gamma(u)} \right]^L.$$  

which is expressed by a simple closed-form representation.

B. Collaborative Detection

It has been extensively shown in the research literature that the performance of energy detection based spectrum sensing can be significantly improved when secondary users collaborate by sharing their information. In this case, the probability of detection and probability of false-alarm for a communication scenario with $n$ collaborating users are given by $Q_d \triangleq 1 - (1 - P_f)^n$ and $Q_f \triangleq 1 - (1 - P_f)^n$, respectively [13]. Based on this, the average probability of detection of a system with no diversity and $\kappa-\mu$ fading in a collaborative scenario $n$—users is deduced by substituting (15) yielding the closed-form representation at the top of the next page. To the best of the Authors’ knowledge, equation (20) is novel.

IV. ENERGY DETECTION OVER $\kappa-\mu$ EXTREME FADING CHANNELS

A. The $\kappa-\mu$ Extreme Fading Model

As already mentioned in Sec. I, the $\kappa-\mu$ Extreme distribution was recently proposed as a fading model that is capable of accounting adequately for severe fading conditions which are mainly encountered in enclosed environments such as airplanes, buses, trains and shopping malls, [24], [25]. Unlike propagation in traditional outdoor and indoor environments, propagation in enclosed environments has been shown to present only a small number of multipath components. To this effect, the use of the Central Limit Theorem (CLT) becomes inappropriate and as a consequence, the corresponding wireless channel can not be accurately characterized by the well known small-scale fading distributions such as Rayleigh, Nakagami—$m$, Hoyt and Weibull [24], [25], [32].

The $\kappa-\mu$ Extreme distribution emerges as a special case of the $\kappa-\mu$ distribution and its normalized PDF is given by [25, eq. (6)], namely,

$$p_\rho(\rho) = 4m^m e^{-2m(1 + \rho^2)} I_1(4m\rho) + e^{-2m} \delta(\rho)$$

where $m = \text{Var}^{-1}(R^2)$ is the Nakagami—$m$ parameter and $\delta(\rho)$ stands for the Dirac delta function [26]. According to [25, eq. (13)] and with the aid of [23, eq. (10)], the corresponding SNR PDF is expressed as follows:

$$p_{\gamma}(\gamma) = \frac{2m}{\sqrt{\gamma}} e^{-2m(1 + \frac{\gamma}{\mu})} I_1(4m\frac{\sqrt{\gamma}}{\mu}) + e^{-2m} \delta(\gamma).$$

It is noted that the above expression will be useful in deriving an analytic expression for the corresponding average probability of detection.

B. Average Detection Probability in $\kappa-\mu$ Extreme Fading

The average detection probability in the presence of $\kappa-\mu$ Extreme fading, $P_d^{\kappa-\mu \text{Ext.}}$, can be derived by following the same procedure as in the derivation of $P_d^{\kappa-\mu}$ in Sec. III. To this end, by substituting (22) into (8), setting $x = \sqrt{2}\gamma$ and noticing that due to the $\delta(\rho)$ function the second term in (22) reduces to zero for $\gamma \neq 0$, one obtains

$$P_d^{\kappa-\mu \text{Ext.}} = \frac{2\sqrt{2m}}{\sqrt{\gamma} e^{2m}} \int_{0}^{\infty} \frac{Q_u(x, \sqrt{\lambda}) I_1(2m\sqrt{\frac{\sqrt{\gamma}}{\mu}} x)}{x^2} \frac{dx}{\sqrt{x}}.$$  

(23)

Notably, the above integral belongs to the same class as the $I_1$ integral. To this effect, by setting $x = \sqrt{2}\gamma$ in (10) and...
\[ Q_{d_{\kappa}} = 1 - \frac{1 - P_{d_{\kappa}}}{P_{d_{\kappa}}} = 1 - \frac{1 - \mu^2 (1 + \kappa) e^{-\mu \kappa}}{(\gamma + \mu + \kappa \mu)^n} \]

substituting in (23) yields the following analytic expression,

\[ T_2 = \sum_{l=0}^{\infty} \frac{\Gamma(l + u, \frac{\lambda}{2})}{2^u \Gamma(l + u)} \int_0^\infty \frac{x^{2l} F_1(l + 1; 2; \frac{4m^2}{\gamma + 2m})}{e^{\frac{x^2}{2}(1 + \frac{2m}{\gamma})}} dx. \]  

(24)

Therefore, by substituting [30, eq. (6.643.2)] into (24) and with the aid of [30, eq. (9.220.2)], the following analytic expression for \( P_{d_{\kappa}} \) is deduced,

\[ P_{d_{\kappa}} = \sum_{l=0}^{\infty} \frac{4m^2 e^{-2m}}{\gamma + 2m} \times \left\{ F_A \left( 1; 1; 2, 1; \frac{1}{1 + \frac{2m}{\gamma}}, \frac{4m^2}{\gamma + 2m} \right) - \frac{\lambda^u}{u!2^u} H_A \left( 1, u; 2, u + 1; \frac{\lambda}{2 + 4m}, \frac{\lambda}{2}, \frac{4m^2}{\gamma + 2m} \right) \right\}. \]  

(25)

Importantly, the above series can be equivalently expressed in closed-form as follows,

\[ P_{d_{\kappa}} = \left\{ F_A \left( 1; 1; 2, 1; \frac{1}{1 + \frac{2m}{\gamma}}, \frac{4m^2}{\gamma + 2m} \right) - \frac{\lambda^u}{u!2^u} H_A \left( 1, u; 2, u + 1; \frac{\lambda}{2 + 4m}, \frac{\lambda}{2}, \frac{4m^2}{\gamma + 2m} \right) \right\}. \]  

(26)

The derivation of (26) is also given in the Appendix.

C. Square-Law Selection (SLS)

As in the case of (SLS) diversity in \( \kappa-\mu \) fading, the average probability for \( \mu \) Extreme fading is obtained by averaging \( P_{d_{\kappa}} \) in [10, eq. (15)] over \( L \) independent \( \kappa-\mu \) Extreme branches. To this effect, by substituting (22) in (16) and utilizing (26), one obtains the following closed-form representation,

\[ P_{d_{\kappa}}^{SLS} = 1 - \prod_{i=1}^{L} \left\{ 1 - F_A \left( 1; 1; 2, 1; \frac{1}{1 + \frac{2m}{\gamma}}, \frac{4m^2}{\gamma + 2m} \right) - \frac{\lambda^u}{u!2^u} H_A \left( 1, u; 2, u + 1; \frac{\lambda}{2 + 4m}, \frac{\lambda}{2}, \frac{4m^2}{\gamma + 2m} \right) \right\}. \]  

(27)

D. Collaborative Detection

It is recalled that when energy detection-based spectrum sensing is performed by \( n \) collaborating users, the corresponding average probability of detection is given by \( Q_d = 1 - (1 - P_d)^n \). Therefore, for the case of \( n \) users collaborating over \( \kappa-\mu \) Extreme fading, the following expression is straightforwardly deduced,

\[ Q_{d_{\kappa}} \]  

(28)

Notably, both (27) and (IV-D) have the same algebraic representation as (18) and (20), respectively.

E. Asymptotic Analysis for small SNR values

One of the major issues in spectrum sensing techniques for cognitive radio based communications is the detection of unknown signals in the low SNR regime. This task is often problematic since for SNR \( \leq 1 \) (linear scale) the detector can not typically distinguish the energy of the signal from the energy of the noise unless the number of samples asymptotically scales as \( 1/SNR^2 \). Furthermore, it has been shown that detection becomes significantly harder below -20dB since the required number of samples becomes \( \geq 10^4 \). In the next Section, it is shown that the \( \kappa-\mu \) Extreme model characterizes acceptably the extreme fading conditions experienced by the detector even at the low SNR regime.

Therefore, it is considered essential to derive asymptotic expressions for \( \gamma \ll 0.3 \) to this end, for \( \gamma \leq -10dB \) i.e \( \gamma \leq 0.1 \) and given that typically \( m \geq 0.5 \), it immediately follows that \( \gamma + 2m \approx 2m \) as well as \( 1 + 2m/\gamma \approx 2m/\gamma \) and \( 2 + 4m/\gamma \approx 4m/\gamma \). Hence, by substituting these tight approximations into (25) and (26) and after some basic algebraic manipulations one obtains,

\[ \lim_{\gamma \to 0} P_{d_{\kappa}} \approx \sum_{l=0}^{\infty} \frac{\gamma^l \Gamma(l + 1; 2; 2; 2m)}{e^{2m} \Gamma(l + u) 2^{-l} m^{-l-1}} = 2m e^{2m} \left\{ F_A \left( 1; 1; 2, 1; \frac{\gamma}{2m}; 2m \right) \right\}. \]  

(29)

The value of \( \gamma \) is in linear scale when the dB notation is omitted.
\[ H_A \left( 1, u; 1, 2, u, u + 1; \frac{\lambda \gamma}{4m}, -\frac{\lambda}{2}, 2m \right) \]

With the aid of the above expression and by recalling that \( P_d^{SLS} = 1 - \prod_{i=1}^{L} \left[ 1 - P_d(\gamma_i) \right] \) and \( Q_d = 1 - (1 - P_d)^n \), asymptotic expressions for small SNR values can be straightforwardly deduced for the case of SLS diversity and collaborative spectrum sensing, namely,

\[
\lim_{\gamma \to 0} P_d^{SLS} = 1 - \prod_{i=1}^{L} \left\{ 1 - \frac{2m}{e^{2m}} F_A \left( 1; 1, 2, 1; \frac{\gamma}{2m}, 2m \right) \right\}
\]

\[
\lim_{\gamma \to 0} Q_d^{\mu, \kappa} = 1 - \left\{ 1 - \frac{2m}{e^{2m}} F_A \left( 1; 1, 2, 1; \frac{\gamma}{2m}, 2m \right) \right\}
\]

respectively.

V. Numerical Results and Discussions

This section is devoted to the analysis of the behaviour of energy detection in \( \kappa - \mu \) and \( \kappa - \mu \times \mu \) Extream fading channels. The corresponding performance is evaluated for different scenarios of interest through both \( P_d \) versus \( \gamma \) curves and complementary receiver operating characteristics (ROC) curves (\( P_m \) versus \( P_f \)). In addition, the effect of the fading parameters \( \kappa \), \( \mu \) and \( m \) on the value of \( P_d \) is numerically quantified.

Fig. 1 demonstrates \( P_d \) vs \( \gamma \) curves for \( \kappa - \mu \) fading for different \( \kappa \) and \( \mu \) values with \( P_f = 0.1 \) and \( u = 2 \). One can observe that the energy detector performs better as \( \kappa \) and \( \mu \) increase due to the higher dominance of the LOS component and the relative advantage of the multipath effect, respectively. For example, for the case of \( \gamma = 15 \) dB and \( \kappa = 1.0 \) (fixed), the \( P_d \) for \( \mu = 0.7 \) is nearly 10% higher than for \( \mu = 0.5 \). In the same context, when \( \mu = 0.7 \) (fixed), the \( P_d \) for \( \kappa = 3.0 \) is 9% higher than for the case of \( \kappa = 1.0 \). As the value of \( \gamma \) decreases to negative values, the effect of varying \( \kappa \) and \( \mu \) is shown to reduce. This can be also observed in Fig. 2 which illustrates complementary ROC curves for the case of a five branch SLS diversity assuming \( u = 2 \), \( \mu = 0.5 \) and \( \kappa = 1.0 \) and average SNR for each branch set to \( \gamma_1 = 0 \) dB, \( \gamma_2 = 1 \) dB, \( \gamma_3 = 2 \) dB, \( \gamma_4 = 3 \) dB and \( \gamma_5 = 4 \) dB. Clearly, the effect of each fading scenario on \( P_m \) is greater as \( \gamma \) increases while the performance of the detector is highly dependent on the number of branches. For example, the value of \( P_m \) for \( L = 1 \) and \( P_f = 0.15 \) (fixed), is approximately 80% larger compared to the corresponding case of \( L = 5 \).

Fig. 3 illustrates the complementary ROC for energy detection with up to eight collaborating users. The fading scenarios considered are the same as in the previous case while the average SNR is set to \( \gamma = 3 \) dB. As expected, the performance of the energy detector improves substantially as the number of users increase.
Regarding the effect of $\kappa-\mu$ Extreme fading conditions, Fig. 4 depicts the behaviour of the energy detector with respect to the corresponding average SNR. It is evident that the performance of the detector is highly dependent upon the severity of fading and it improves substantially as $m$ increases. Indicatively, by increasing the fading severity from $m = 0.7$ to $m = 0.5$ for $\gamma = 10$ dB and $P_f = 0.1$ results to a 16% performance reduction.

In the same context, Fig. 5 demonstrates the significant performance improvement when the SLS diversity scheme is employed. For $P_f = 0.2$, the value of $P_m$, over the single branch operation, reduces by about 30% and 58% for $L = 2$ and $L = 3$, respectively, for both $m = 0.6$ and $m = 1.8$.

In the case of collaborating spectrum sensing with up to four users for $u = 2$, $\gamma = 3$ dB, the value of $Q_m$ reduces as the number of users increases for any value of $m$. For example, Fig. 6 illustrates that for a single user detection for $P_f = 0.2$ and $m = 0.6$, the value of $P_m$ is 67% higher compare to the case that $n = 4$.

It is also important to quantify the effect of the fading parameters on the system performance. Although it is undoubtedly elucidating to carry out this task analytically, this is unfortunately impossible due to the high algebraic intractability of the involved mathematical representations. As a result, this effect is only analyzed numerically. To this end, Fig. 7 depicts the behaviour of $P_d$ versus $\kappa$ for $P_f = 0.1$, $u = 3$, $\mu = 0.2$ and different values of $\gamma$. One can observe the significant deviation of the $P_d$ even for small variations of $\kappa$ and/or $\gamma$. For example, for $\gamma = 13$ dB, it is shown that $P_d = 0.55$ and $P_d = 0.675$ for $\kappa = 1$ and $\kappa = 4$, respectively. Furthermore, for $\kappa = 8$, $P_d = 0.47$ for $\gamma = 5$ dB and $P_d = 0.67$ for $\gamma = 9$ dB. Likewise, the behavior of $P_d$ versus $\mu$ is illustrated in Fig. 8 for $P_f = 0.1$, $u = 3$, $\gamma = 2$ and different values of $\gamma$. Clearly, for $\gamma = 10$ dB, it is shown that $P_d = 0.52$ and $P_d = 0.815$ for $\mu = 0.2$ and $\mu = 1.0$, respectively. Also, for $\mu = 0.8$, one obtains $P_d = 0.55$ for $\gamma = 6$ dB and $P_d = 0.88$ for $\gamma = 13$ dB.

In the same context, it is important to note that the $\kappa-\mu$ Extreme fading model can provide adequate characterization of the fading effect in the low SNR regime. It is recalled here that energy detection in $\gamma = -20$ dB is a challenging task since the detector can not distinguish unknown signals with low power from noise power. Fig. 9, depicts the behaviour of the energy detector in $\kappa-\mu$ Extreme fading conditions with respect to the fading severity and SLS diversity. It is clearly seen that the value of $P_m$ is affected significantly by the value of $m$ while it decreases substantially as the number of diversity branches increases. For example, for $P_f = 0.3$, $u = 2$, $\gamma = -20$ dB and $L = 6$, it is seen that $0.18 \leq P_m \leq 0.28$ for $0.5 \leq m \leq 1$ and $P_m < 0.17$ for $m > 1$. For larger numbers of branches ($L > 6$), the value of $P_m$ continues to reduce but at a much smaller pace and thus a small benefit will be gained at the expense of significantly higher complexity.

Finally, the dramatic effect of the different fading conditions modeled by the $\kappa-\mu$ and $\kappa-\mu$ Extreme distributions is also demonstrated through comparisons with results from existing works. For example, for $\gamma = 10$ dB and $u = 2$, $P_f = 0.1$ in Fig. 1, one obtains $P_d = 0.77$ for Rayleigh fading in [10],
Expressions were derived for the average probability of detection for both cases. The overall performance of the detector is largely affected by the value of the corresponding fading parameters since it is very sensitive even at small variations, particularly as the average SNR increases. It was also demonstrated that a significant performance improvement is achieved in both severe and moderate fading conditions as the number of users or diversity branches increases. Furthermore, it was shown that the $\kappa-\mu$ Extreme fading model provides adequate fading characterization of the fading effect in the low SNR regime and thus improves the performance of the energy detector. As a result, the offered results are useful in quantifying the effect of fading in energy detection spectrum sensing which can ultimately lead to improved and/or more energy efficient cognitive radio-based communication systems.

**APPENDIX**

According to [30, eq. (8.354.2)], the upper incomplete gamma function can be expressed as $\Gamma(a, x) \triangleq \Gamma(a) - \sum_{n=0}^{\infty} \frac{(-1)^n x^{a+n}}{n!(a+n)}$. By also recalling that $\Gamma(a+1) \triangleq a!$ and $1F_1(a; b; x) \triangleq \sum_{n=0}^{\infty} \frac{(a)_n x^n}{(b)_n n!}$ and performing the necessary change of variables, equation (15) can be expressed as follows:

$$
\mathcal{P}_{d\kappa-\mu} = \frac{\sum_{l=0}^{\infty} \sum_{j=0}^{\infty} \mu^{l+2j}(1+\kappa)^{\mu+2j} \gamma^l \Gamma(\mu + l + j) \kappa^l \gamma^l \Gamma(\mu + l + j)}{(l + i + u)(\gamma + \mu + \kappa) \mu^{l+2j}} \quad (A.1)
$$

By recalling that the Pochhammer symbol is defined as $(a)_n \triangleq \Gamma(a + n)/\Gamma(a)$, the Gamma functions in $\mathcal{I}_3$ and $\mathcal{I}_4$ can be expressed as $\Gamma(\mu + l + j) = (\mu)_l \Gamma(\mu)$, $\Gamma(\mu + l + j) = (\mu)_l \Gamma(\mu)$, $\Gamma(l + u) = (u)_l \Gamma(u)$ and $\Gamma(l + u) = (u)_l \Gamma(u)$ and $\Gamma(l + u) = (u)_l \Gamma(u)$. Furthermore, the $(l + i + u)$ term in $\mathcal{I}_3$ can be re-written as,

$$
(l + i + u) = \frac{(l + i + u)!}{(l + i + u - 1)!} = \frac{\Gamma(l + i + u + 1)}{\Gamma(l + i + u)} \quad (A.2)
$$

By expressing each Gamma function in terms of the Pochhammer symbol it follows that,

$$
(l + i + u) = \frac{(u + 1)_l \Gamma(u + 1)}{(u)_l \Gamma(u)} = \frac{(u + 1)_l (u + 1)_l}{(u)_l (u)_l} \quad (A.3)
$$

By substituting accordingly in $\mathcal{I}_3$ and $\mathcal{I}_4$, re-writing $(u)_1 = (u)/((u) - 1)! = u$ and carrying out some long but basic algebraic manipulations one obtains,

$$
\mathcal{P}_{d\kappa-\mu} = \frac{\mu^\mu(1+\kappa)^{\mu+\gamma} \kappa^\gamma}{(\gamma + \mu + \kappa)\mu} \sum_{l=0}^{\infty} \sum_{j=0}^{\infty} \frac{(\mu)_l (\kappa(1+\kappa)\mu^2)^j (\gamma + \mu + \kappa)\mu}{l!} \quad (A.4)
$$

**VI. CONCLUSION**

This work analyzed the performance of energy detection in $\kappa-\mu$ and $\kappa-\mu$ Extreme fading channels. Novel analytic
Fig. 9. \( P_m \) vs \( m \) for \( \kappa-\mu \) Extreme fading with \( P_f = 0.3 \), \( u = 2 \) and \( \gamma = -20 \) dB and \( L \) diversity branches.

Importantly, the algebraic form of \( I_6 \) s the same as the representation of the Lauricella hypergeometric function in [30, eq. (9.19)], [36, eq. (1)]. Similarly \( I_6 \) can be expressed in terms of the complete triple hypergeometric function of the first kind in [35, eq. (1.1)]. To this effect, by performing the necessary variable transformation, equation (15) is deduced.

It is noted that (26) has the same algebraic form as (15) and therefore, it can be derived by following the same procedure.

**ACKNOWLEDGEMENTS**

The Authors would like to thank the Associate Editor and the Anonymous Reviewers for their constructive comments and suggestions which improved the quality of this work.

**REFERENCES**


[36] H. M. Srivastava, “The same as the...


Paschalis C. Sofotasios (S’06, M’10) was born in Volos, Greece in 1978. He received the M.Eng. degree in Electronic and Communications Engineering from the University of Newcastle upon Tyne, UK, the M.Sc. degree in Satellite Communications Engineering from the University of Surrey, UK and the Ph.D. degree in Electronic and Electrical Engineering from the University of Leeds, UK. Since October 2010, he has been a Research Fellow at the University of Leeds and during Fall 2011 he was a Visiting Researcher at the CORES Lab of the University of California, Los Angeles (UCLA). His research interests are in the area of wireless communications with emphasis on characterization and modelling, cognitive radio, cooperative systems and free-space-optical communications.

Eric Rebeiz (S’09) received his B.S. degree (summa cum laude) from the University of Massachusetts Amherst in 2008 and his M.S. degree from the University of Southern California in 2009, both in Electrical Engineering. He is currently a Ph.D. Candidate at the University of California Los Angeles advised by Professor Danijela Cabric. His current research interests pertain to the field of cognitive radios and include low power cyclostationary-based spectrum sensing, compressive sensing, and modulation classification in wideband channels.

Li Zhang (S’99, M’03, SM’12) received her Ph.D. degree in Communications from the University of York, York, UK in 2003. Since September 2004, she has been with the School of Electronics and Electrical Engineering at the University of Leeds, where she became senior lecturer in 2011. Her general research interests are in the area of signal processing for wireless and power line communications.

Theodoros Tsiftsis (S’02, M’04, SM’10) was born in Lamia, Greece, in 1970. He received the degree in physics from the Aristotle University of Thessaloniki, Greece, in 1993, and the M.Sc. degree in digital systems engineering from the Heriot-Watt University, Edinburgh, Scotland, U.K., in 1995. Also, he received the M.Sc. degree in decision sciences from the Athens University of Economics and Business, Greece, in 2000 and the Ph.D. degree in electrical engineering from the University of Patras, Greece, in 2006. He is currently an Assistant Professor in the Department of Electrical Engineering at Technological Educational Institute of Lamia, Greece. His main research interest is concerned with advanced analogue and digital signal processing for ultrasonic instrumentation and wireless communication systems. He teaches digital signal processing, microcontrollers/microprocessors, VLSI and embedded systems design, hardware description languages at both undergraduate and postgraduate level. Dr. Tsiftsis acts as reviewer for several international journals and he is member of the Editorial Boards of *IEEE Transactions on Communications* and *IEEE Communications Letters*.

Steven Freear (S’95, M’97, SM’11) gained his doctorate in 1997 and subsequently worked in the electronics industry for 7 years as a VLSI system designer. He was appointed Lecturer (Assistant Professor) and then Senior Lecturer (Associate Professor) in 2006 and 2008 respectively at the School of Electronic and Electrical Engineering at the University of Leeds. His main research interest is concerned with advanced analogue and digital signal processing for ultrasonics, Ferroelectrics and Frequency Control (UFFC) and the International Journal of Electronics. He will become Editor-in-Chief of the *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control (UFFC)* from 2013.

Danijela Cabric received the Dipl. Ing. degree from the University of Belgrade, Serbia, in 1998, and the M.S. degree in electrical engineering from the University of California, Los Angeles, in 2001. She received her Ph.D. degree in electrical engineering from the University of California, Berkeley, in 2007, where she was a member of the Berkeley Wireless Research Center. In 2008, she joined the faculty of the Electrical Engineering Department at the University of California, Los Angeles as an Assistant Professor. Her key contributions involve the novel radio architecture, signal processing, and networking techniques to implement spectrum sensing functionality in cognitive radios. She has written three book chapters and over seventy major journal and conference papers in the fields of wireless communications and circuits and embedded systems. She was awarded Samuei Fellowship in 2008, and Okawa Foundation research grant in 2009.