Decentralized Admission Control and Resource Allocation for Power-Controlled Wireless Networks

Sławomir Stańczak$^{1,2}$

joint work with

Holger Boche$^{1,2}$, Marcin Wiczanowski$^1$ and Angela Feistel$^2$

$^1$Fraunhofer German-Sino Lab for Mobile Communications (MCI)
Berlin, Germany

$^2$Heinrich Hertz Chair for Mobile Communications
Faculty of EECS
Technical University of Berlin

22 September 2009
Outline

1. Introduction

2. Physical-Layer Abstraction by Interference Functions

3. User-centric Approaches

4. Network-centric approaches
   - Distributed Power Control Algorithms
   - Incorporating QoS requirements
Outline

1. Introduction

2. Physical-Layer Abstraction by Interference Functions

3. User-centric Approaches

4. Network-centric approaches
   - Distributed Power Control Algorithms
   - Incorporating QoS requirements
Wireless Networks

- **Goal:** Study resource allocation and interference management
- **Focus:** High data rates, low or moderate channel dynamics
  - Energy supply is not a bottleneck.
  - Wireless mesh networks, cellular networks
Wireless Channel Characteristics

- Radio propagation channel is unreliable.
  - channel fading, path loss, channel conditions are time-varying ...
- Power and bandwidth are limited.
- Wireless spectrum is a shared medium.
  - Link capacities are elastic.
  - Network cannot be regarded as a collection of point-to-point links.
  - Performance is maximized by tolerating interference in a controlled way.

Resource allocation and interference management are necessary.
Wireless resources: power, time, frequency, space, codes, routes...

Mechanisms for resource allocation and interference management
- Multiple antenna techniques
- MAC: power control and scheduling
- routing
- ...

Cross-layer protocols
Applications

- Voice transmission
  - Inelastic traffic: QoS requirements need to be satisfied permanently.
- Data applications (WWW browsing, e-mail, ftp)
  - Low QoS levels are temporarily acceptable.
  - Elastic traffic: Applications modify their data rates according to available resources in communication networks.
Quality of Service

- User-centric approaches (inelastic applications):
  - Satisfy strict QoS requirements of applications permanently.

- Network-centric approaches (elastic applications):
  - Maximize the aggregate utility as perceived by the network operator.
  - Address the issue of fairness.

![Diagram showing Feasible QoS region, minimum total power, max-min fairness, and weighted sum optimization criteria.]

\[
\max \sum_k \alpha_k QoS_k \quad \text{(weighted sum optimization)}
\]

\[
\max \sum_k QoS_k \quad \text{(best overall efficiency)}
\]
The focus of this talk

- Power control with some aspects of the physical-layer design.

- Single-hop communication with $K > 1$ logical links (users)
- Concurrent transmission (works with any scheduling protocol).
- Each user is decoded (single-user decoding).
- Combination with routing and network coding strategies possible.
Outline

1. Introduction

2. Physical-Layer Abstraction by Interference Functions

3. User-centric Approaches

4. Network-centric approaches
   - Distributed Power Control Algorithms
   - Incorporating QoS requirements
Given a channel, $F_\gamma$ is the set of all QoS values that can be achieved by means of power control with all links being active concurrently.

- **Assumption:** $\omega_k \uparrow \iff \text{QoS} \uparrow$
  - Downward-comprehensive
  - Upper-bounded
- may be non-convex.
- depends on power constraints $P$.

- $F_\gamma$ depends on the physical-layer realization: Key properties of many multiuser systems are captured by interference functions.
Signal-to-Interference(+noise) Ratio (SIR)

Strictly monotonic QoS-SIR map: \( \gamma : \mathbb{R} \rightarrow \mathbb{R}_+ \)

For any \( \omega \in F_\gamma \), there is a power vector \( p \in P \) such that

\[
\gamma(\omega_k) = \text{SIR}_k(p) = \frac{p_k}{l_k(p)}
\]

\( \leftarrow \) transmit power

\( \leftarrow \) interference function

- e.g. Gaussian capacity (in nats/channel use): \( \gamma(x) = e^x - 1, x \geq 0 \).
Axiomatic Interference Function

Standard Interference Functions (SIF), Yates’95

A1 Positivity: \( I_k(p) > 0 \) for all \( p \geq 0 \).

A2 Scalability: \( I_k(\mu p) < \mu I_k(p) \) for any \( p \geq 0 \) and for all \( \mu > 1 \).

A3 Monotonicity: \( I_k(p^{(1)}) \geq I_k(p^{(2)}) \) if \( p^{(1)} \geq p^{(2)} \).

- It models many practical interference scenarios.
**Interference Function: Examples**

**Linear interference function**

\[ I_k(p) = (Vp + z)_k \]

- Matched-filter receiver
- SIC receiver

**Minimum interference function**

\[ I_k(p) = \min_{u_k \in U_k} (V(u)p + z(u))_k \]

- MMSE receiver

---

**Graphical Illustration**

- QoS levels 1, 2, 3
- Interference paths
- SIR user 1, SIR user 2

---

UCLA, 22.09.2009
Outline

1. Introduction
2. Physical-Layer Abstraction by Interference Functions
3. User-centric Approaches
4. Network-centric approaches
   - Distributed Power Control Algorithms
   - Incorporating QoS requirements
Problem Statement

Problem (QoS-based power control under SIFs)

\[ p(\omega) = \arg \min_{p \in P(\omega)} w^T p \quad \text{w} > 0 \]

\[ P(\omega) := \{ p \in \mathbb{R}_+^K : \forall k \text{ SIR}_k(p) \geq \gamma(\omega_k) \} . \]

Minimum total power

Feasible QoS region

Valid power set

\[ p_1 = \gamma_1 l_1(p) \]

\[ p_2 = \gamma_2 l_2(p) \]

Zander’92, Foschini’94, Yates’95, Ulukus’98, Bambos’00, Boche&Schubert …
If $\omega$ is feasible, then the concurrent iterations

$$\forall k \ p_k(n + 1) = \gamma_k l_k(p(n))$$

converge to $p(\omega)$, where $\gamma_k \equiv \gamma(\omega_k)$.

- Component-wise increasing (decreasing) if $p(0) = 0 \ (p(0) \in P(\omega))$.
- Amenable to distributed implementation, scalable, works for any SIF

But how should new users join the network without disrupting the connections of active users?
If $\omega$ is feasible, then the concurrent iterations

$$\forall_k p_k(n + 1) = \gamma_k I_k(p(n))$$

converge to $p(\omega)$, where $\gamma_k \equiv \gamma(\omega_k)$.

- Component-wise increasing (decreasing) if $p(0) = 0$ ($p(0) \in P(\omega)$).
- Amenable to distributed implementation, scalable, works for any SIF
- But how should new users join the network without disrupting the connections of active users?
User $k$ is called active at time $n$ if $\text{SIR}_k(p(n)) \geq \gamma_k$.

Define $\mathcal{A}_n$ to be the set of all active users at time $n$ and $\mathcal{B}_n := \mathcal{K} \setminus \mathcal{A}_n$.

**Power control with active link protection ($\delta > 1$)**

\[
p_k(n + 1) = \begin{cases} 
\delta \gamma_k l_k(p(n)) & k \in \mathcal{A}_n \text{ (active users)} \\
\delta p_k(n) = \delta^{n+1} p_k(0) & k \in \mathcal{B}_n \text{ (inactive users)}
\end{cases}
\]

- $\delta > 1$ can be interpreted as protection margin.
  - the larger $\delta$, the faster power-up of the inactive users.
  - $\delta$ cannot be too large for all users to be fully admissible.

Bambos'00, Chee Wei Tan'09 (only $l_k(p) = (Vp + z)_k$)
Properties of the admission control scheme

**Theorem**

Let $I_k$ be any standard interference function. Then,

- All SIRs converge to some values.
- All users are admitted in finite time if $\gamma = (\gamma_1, \ldots, \gamma_K)$ is feasible.
- Transmit powers are bounded if and only if $\delta \cdot \gamma$ is feasible.
- Active users ($k \in A_n$):
  - Preservation of active users: $A_n \subseteq A_{n+1}$.
  - Bounded power overshoot: $p_k(n+1) < \delta p_k(n)$.
- Inactive users ($k \in B_n$):
  - SIRs of inactive users are increasing $\text{SIR}_k(p(n)) < \text{SIR}_k(p(n+1))$.

Stanczak&Kaliszan&Bambos'09
No power constraints, TX and RX beamforming

\[ K = 10, \quad n_T = n_R = 4, \quad \gamma = 8, \quad \delta \gamma = 9.6, \quad A_n = \{1, \ldots, 5\} \]

\[ \text{The highest feasible SIR (example):} \]
- 0.88 (fixed beamformers), 1.37 (RX beamforming), 8 (TX/RX beamforming)
Theorem

Suppose that $\delta \gamma$ is feasible and

$$p(m) \leq \beta \delta I(p(m))$$

holds for some $m \in \mathbb{N}_0$ and $\beta \in [1, \beta_{\text{max}}]$. Then, there exists $\beta_{\text{max}} > 1$ such that $A_n \subseteq A_{n+1}$ for all $n \geq m$.

- Active users send distress signals until the condition is satisfied.

Stanczak & Kaliszan & Bambos'09
1. Introduction

2. Physical-Layer Abstraction by Interference Functions

3. User-centric Approaches

4. Network-centric approaches
   - Distributed Power Control Algorithms
   - Incorporating QoS requirements
Problem Statement

Problem (Utility-based power control)

\[ \omega^* = \arg \max_{\omega \in F_\gamma} \mathbf{w}^T \omega \quad \mathbf{w} > 0. \]

\( \omega^* \) is a maximal point of \( F_\gamma \)

Feasible QoS region \( F_\gamma \)

Maximal point of \( F_\gamma \)

Goodman’00, Saraydar’02, Xiao’03, Neely’03, Chiang’04, Huang’05, Tassiulas’05 …
Convexity of Feasible QoS Region

Find a class of strictly increasing and concave utility functions \( \Psi \) with

\[
\gamma(\Psi(x)) = x, \ x \geq 0
\]

so that \( F_\gamma \) is a convex set.
Theorem (Convexity under a Linear Interference Function)

If $\gamma$ with $\gamma(\Psi(x)) = x$, $x \geq 0$, is log-convex, then the feasible QoS region is a convex set, regardless of the type of power constraints.

- **Observation:** $\gamma$ is log-convex if and only if $\Psi_e(x) := \Psi(e^x)$ is concave.

- **Further related results:**
  - Log-convexity of $\gamma(x)$ is necessary for the region to be convex in general.
  - Convexity of $\gamma(x)$ is sufficient if $V$ is confined to belong to some subset of nonnegative matrices.
Examples of Function Classes

\[ \Psi_\alpha(x) = \begin{cases} \frac{x^{1-\alpha}}{1-\alpha} & \alpha > 1 \\ \log(x) & \alpha = 1 \end{cases} \quad \tilde{\Psi}_\alpha(x) = \begin{cases} \log x & \alpha = 1 \\ \log \frac{x}{1+x} & \alpha = 2 \\ \log \frac{x}{1+x} + \sum_{j=1}^{\alpha-2} \frac{1}{j(1+x)^j} & \alpha > 2 \end{cases} \]

- \( \alpha = 1 \): Throughput maximization
- \( \alpha \to \infty \): Max-min fairness
Arbitrarily Close Approximation of Max-Min Fairness

Let $\omega^*_k = \Psi_\alpha(SIR^*_k)$ and let $\nu^*_k = \log(1 + SIR^*_k)$. Then, $\nu^*$ converges to the max-min rate allocation as $\alpha \to \infty$. 

![Graph showing flow rates and max-min rate allocation](image-url)
Efficiency of the Max-Min SIR Power Allocation

Theorem

If $\bar{p}$ and $\bar{q}$ are positive right and left eigenvectors of $B^{(k_0)} = V + \frac{1}{P_{k_0}}ze^T$, then

(i) $p$ is max-min fair power allocation if and only if $p = \bar{p}$.
(ii) $\omega^*$ is max-min fair $\bar{\omega}$ if and only if $w = w^* = \bar{q} \circ \bar{p} > 0$. 

\[ \alpha_2 < \alpha_3 < \alpha_4 \]

$\omega_1 = \log(\text{SIR}_1)$

$\omega_2 = \log(\text{SIR}_2)$
Alternating optimization

1. Given $U(t - 1)$ compute $p(t)$

2. Given $p(t)$ compute $U(t)$
Joint Power and Receiver Control

Alternating optimization

1. Given $U(t-1)$ compute $p(t)$
   (i) Compute the weight vector: $w = y(B^{(m)}) \circ x(B^{(m)})$, $m = \arg\max_k \rho(B^{(k)})$
   (ii) Compute the QoS vector: $\omega^* = \arg\max_{\omega \in F} w^T \omega$
   (iii) $p(t) = (I - \Gamma(\omega^*)V)^{-1}\Gamma(\omega^*)z$, $\Gamma(\omega) = \text{diag}(\gamma(\omega_1), \ldots, \gamma(\omega_K))$

2. Given $p(t)$ compute $U(t)$
   (i) $\forall_k u_k(t) = \arg\max_{\|x\|_2 = 1} \text{SIR}_k(p(t), x)$
Alternating optimization

1. Given $U(t-1)$ compute $p(t)$
   (i) Compute the weight vector: $w = y(B^{(m)}) \circ x(B^{(m)})$, $m = \arg \max_k \rho(B^{(k)})$
   (ii) Compute the QoS vector: $\omega^* = \arg \max_{\omega \in F_{\gamma}} w^T \omega$
   (iii) $p(t) = (I - \Gamma(\omega^*)V)^{-1} \Gamma(\omega^*)z$, $\Gamma(\omega) = \text{diag}(\gamma(\omega_1), \ldots, \gamma(\omega_K))$

2. Given $p(t)$ compute $U(t)$
   (i) $\forall_k u_k(t) = \arg \max_{\|x\|_2=1} \text{SIR}_k(p(t), x)$

- monotonic convergence to max-min SIR-balancing solution
- **But:** not amenable to distributed implementation
- Theory provides a basis for novel decentralized algorithms for finding a saddle point of the aggregate utility function.
Some Simulation Results

- **Minimum SIR**
  - Red: Maximum power
  - Blue: PC
  - Green: PC + RX Beamf.
  - Purple: PC + RX/TX Beamf.

- **Average Delay vs Arrival Rate**
  - Red: Static
  - Blue: PC (Utility-Max.)
  - Green: PC (Queue-Weighted Utility-Max.)
  - Purple: PC + Beamforming (Max-Min-SINR)

- **Utility-Max.**
  - Blue: PC (Utility-Max.)
  - Green: PC (Queue-Weighted Utility-Max.)

UCLA, 22.09.2009
Outline

1 Introduction

2 Physical-Layer Abstraction by Interference Functions

3 User-centric Approaches

4 Network-centric approaches
   - Distributed Power Control Algorithms
   - Incorporating QoS requirements
Utility-Based Power Control

Equivalent minimization problem: $\psi(x) = -\Psi(x)$

$$p^* = \arg \min_{p \in P} F(p) = \arg \min_{p \in P} \sum_k w_k \psi(\text{SIR}_k(p)).$$

- Positivity of minimizers: $p^* > 0$
- Even if $\psi(e^x)$ is convex, the problem is not convex in general.
Theorem

If $I_k(e^s)$ is log-convex and $\psi(e^x)$ convex, the following problem is convex:

$$s^* = \arg \min_{s \in S} F_e(s)$$

$$\begin{align*}
S & := \{\log x : x \in P_+\} \\
F_e(s) &= F(e^s)
\end{align*}$$

- $I_k(e^s) = \sum_l v_{k,l} e^{s_l} + z_k$ is log-convex (Hoelder inequality).
Gradient-Projection Algorithm

- Let $\tau > 0$ be constant step size (small enough), and let

$$s(n + 1) = \Pi_S \left[ s(n) - \tau \nabla F_e(s(n)) \right] \quad s(0) \in S$$

- $\nabla F_e(s) = \text{diag}(e^{s_1}, \ldots, e^{s_K}) \nabla F(e^s)$:

$$\nabla F(p) = (I - V^T \Gamma(p))g(p)$$

- $g_k(p) = w_k \psi'(\text{SIR}_k(p))\text{SIR}_k(p)/p_k$ (locally available)

- $\Gamma(p) = \text{diag}(\text{SIR}_1(p), \ldots, \text{SIR}_K(p))$
Min-max reformulation

\[
\min_{s} \max_{u} \sum_{k} w_k \psi \left( \frac{e^{s_k}}{u_k} \right) \quad \text{subject to } \begin{cases} 
e^{s} - \hat{p} \leq 0 \\
u - t \leq 0 \
\forall_k l_k(e^{s}) - t_k = 0.
\end{cases}
\]

- Linear interference function: \( l_k(e^{s}) = (V e^{s} + z)_k \).
- The Hessian is diagonal and its diagonals are given by the gradient.
Conditional Newton Algorithm

\[
\begin{align*}
\begin{pmatrix} s(n+1) \\ \mu(n+1) \end{pmatrix} &= \begin{pmatrix} s(n) \\ \mu(n) \end{pmatrix} - \left( \nabla^2_{(s,\mu)} L(z(n)) \right)^{-1} \nabla_{(s,\mu)} L(z(n)) \\
\nabla_{(u,\lambda^u,\lambda,t)} L(z(n+1)) &= 0
\end{align*}
\]

can be solved explicitly

\[ L(z) = L(s, u, \mu, \lambda^u, \lambda, t) \]: A modified Lagrangian function.

- Quadratic convergence.
- Global convergence if \( \psi(x) = -\log(x) \) and \( \psi(x) = 1/x \).
- No step size.
- Distributed implementation possible!

K = 50
\( \psi(x) = -\log(x) \)
- - - - - Conditional Newton
- - - - - - - Gradient projection
Adjoint Networks: A Simple Example

Primal network:
- Measure SIR at E1 and E2
- E1 and E2 compute some messages based on local measurements/parameters
Adjoint Networks: A Simple Example

Primal network:
- Measure SIR at E1 and E2
- E1 and E2 compute some messages based on local measurements/parameters
Adjoint Networks: A Simple Example

Primal network:
- Measure SIR at E1 and E2
- E1 and E2 compute some messages based on local measurements/parameters
Adjoint Networks: A Simple Example

Primal network:
- Measure SIR at E1 and E2
- E1 and E2 compute some messages based on local measurements/parameters
Adjoint Networks: A Simple Example

Reversed network:
- The messages determine the transmit powers of E1 and E2.
- Cooperation by interference

⇒ S1 and S2 estimate the messages from the received powers

- Significant gains compared to schemes relying on flooding protocols.
- Estimation errors are dealt with stochastic approximation.
Reversed network:

- The messages determine the transmit powers of E1 and E2.
- **Cooperation by interference**

⇒ S1 and S2 estimate the messages from the received powers

- Significant gains compared to schemes relying on flooding protocols.
- Estimation errors are dealt with stochastic approximation.
Reversed network:

- The messages determine the transmit powers of E1 and E2.
- **Cooperation by interference**

⇒ S1 and S2 estimate the messages from the received powers

- Significant gains compared to schemes relying on flooding protocols.
- Estimation errors are dealt with stochastic approximation.
Adjoint Networks: A Simple Example

Reversed network:

- The messages determine the transmit powers of E1 and E2.
- Cooperation by interference

⇒ S1 and S2 estimate the messages from the received powers

- Significant gains compared to schemes relying on flooding protocols.
- Estimation errors are dealt with stochastic approximation.
Outline

1 Introduction

2 Physical-Layer Abstraction by Interference Functions

3 User-centric Approaches

4 Network-centric approaches
   - Distributed Power Control Algorithms
   - Incorporating QoS requirements
Problem Statement

\[ s^*(\omega) := \arg \min_{s \in S} F_e(s) \quad \text{s.t.} \quad \forall_k f_k(s) := l_k(e^s)/e^{s_k} - 1/\gamma_k \leq 0 \]

- The projection is not amenable to distributed implementation.
Primal-dual algorithm based on standard Lagrangian

Primal-dual iteration under individual power constraints

\[
\begin{aligned}
    s_k(n + 1) &= \min \left\{ s_k(n) - \delta e^{s_k(n)} \left[ h_k(s(n)) + \sum \lambda_k(s(n), \mu(n)) \right], \log(P_k) \right\} \\
    \lambda_k(n + 1) &= \max \left\{ 0, \lambda_k(n) + \delta f_k(s(n)) \right\}
\end{aligned}
\]

\[\Sigma_k(s, \lambda) = \sum_l v_{l,k} \left( \frac{\lambda_l}{e^{s_l}} + |\text{SIR}_l(e^s)g_l(s)| \right) = \sum_l v_{l,k} m_l(s, \mu_l)\]

- Estimation of \(\Sigma_k\) using the adjoint network.

UCLA, 22.09.2009
Fraunhofer German−Sino Lab Mobile Communications
Soft QoS Support

\[
\tilde{F}_\alpha(z) = \sum_{k \in A} a_k \psi_\alpha \left( \frac{\text{SIR}_k(p)}{\gamma_k} \right) + \sum_{k \in B} b_k \psi(SIR_k(p)) .
\]

- **\(A\)**: QoS users need to satisfy \(\text{SIR}_k \geq \gamma_k, k \in A\)
- **\(B\)**: best-effort users
  - **\(A \setminus B\)**: pure QoS users (voice)
  - **\(A \cap B\)**: best-effort users with QoS requirements (video)
  - **\(B \setminus A\)**: pure best-effort users (data)

- Each user, say user \(k\), determines its utility by choosing \(\alpha_k \geq 1\).
- Slightly modified algorithms
- Amenable to distributed implementation
Soft QoS Support: Example

1 - max-min-fairness

2 - Utility-based power control with $\alpha_2 = 1$

3 - Utility-based power control with soft QoS support, $\alpha_2 \to \infty$

- No overshoot of user 2.
Conclusions and Outlook

- The power control problem is relatively well-understood.
- Throughput maximization in the low SINR regime is an open problem.

Outlook

- Joint optimization of
  - transmit powers,
  - schedulers (time+frequency),
  - physical-layer (single- and multi-mode transmission).
- Dynamic optimization over finite and infinite time horizon.
- Stochastic power control.
- ...