Using the Time Dimension to Sense Signals with Partial Spectral Overlap

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Outline

- Goal, Motivation, and Existing Work
- System Model
  - Assumptions
  - Time-Frequency Map
- Proposed Algorithm: NNMF-based Algorithm
- Novel Performance Metrics: Why and How
- Simulation Results
- Conclusions and Future Work
Distinguishing Signals with Spectral Overlap

That is,

- Counting number of signals received
- Detecting sets of discrete Fourier transform bins occupied by each signal
Potential Applications

Spectral overlap by design

- IEEE 802.11b/g channels in 2.4GHz
- Channel bonding in IEEE 802.11n


Lack of Guard Bands

- IEEE 802.11n in 5GHz bands
- LTE-Advanced


Motivation

Improved sensing accuracy


Multi-signal Classification

Multichannel Traffic Estimation and Prediction

Signal 1: DSSS
Signal 2: 4-QAM
Signal 3: OFDM

Magnitude (dB)
Frequency (MHz)

Occupied
Unoccupied
### Existing Work

<table>
<thead>
<tr>
<th>Based on</th>
<th>Blind</th>
<th>Single Antenna</th>
<th>Spectral Overlap</th>
<th>Detect Bands</th>
<th>Blind to Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission protocols [4-5]</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Cyclic frequency [7]</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
</tr>
<tr>
<td>Channel model &amp; location [6]</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>Angle of Arrival [8]</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>Random Matrix Theory [9]</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
</tr>
<tr>
<td>Multiple CRs [10-11]</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
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<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Multiple Power Spectrum Measurements</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

**Proposed method**
### System Model

#### Incumbent Users
- $M$ transmitters with center frequency $F_m$ and bandwidth $W_m$
  - Power spectrum received from $m^{th}$ transmitter: $\Sigma_m$
  - Activity $a_m[t] = 1$ if transmitting at time $t$, 0 otherwise

#### Wideband sensor
- Baseband bandwidth $W$ Hz, known noise power $\sigma_v^2$
- Welch power spectrum estimator using FFT of length $F$
- can store multiple power spectrum measurements

#### Received power spectrum:
$$Y[t] = \sum_{m=1}^{M} a_m[t] \Sigma_m + \nu[t]$$

- Estimated energy received from $m^{th}$ transmitter
- Estimated noise energy
Time-Frequency Map

- Time-Freq. map $E$ of received energy: $E = [Y[1], Y[2], \ldots, Y[T]]^T$
- Define matrices: $A_{tm} = a_m[t]$, $\Sigma_{mf} = \Sigma_m(f)$, and $\Delta_{tf} = v[t](f)$

$$E = A\Sigma + \Delta$$

**Example:** $M = 3$, $F = 512$, $T = 30$

- **Input:** Power Spectrum measurements
  - Output computed by Non-Negative Matrix Factorization (NNMF)
- **Output:** Time-Freq of Each Tx
  - $A(1) \times \Sigma(1)$
  - $A(2) \times \Sigma(2)$
  - $A(3) \times \Sigma(3)$
Non-Negative Matrix Factorization (NNMF)

- Let $\hat{M} = \text{Estimated number of received signals}$
- NNMF finds $\hat{A} \in \mathbb{R}^{T \times \hat{M}}$, $\hat{\Sigma} \in \mathbb{R}^{\hat{M} \times F}$ to minimize $\|E - \hat{A}\hat{\Sigma}\|_F^2$

Challenges:

- Estimating $\hat{M}$ is hard when $T < F$

- Non-convex cost function
  - $\Rightarrow$ convergence to global minima not guaranteed

- Cost function is not probabilistic
  - $\Rightarrow$ Not robust to noise

- Non-unique solution and $\hat{A}$ is not binary
  - $\Rightarrow$ $\hat{\Sigma} \neq \Sigma$, i.e., thresholding $\hat{\Sigma}$ will not detect all occupied DFT bins
Non-Negative Matrix Factorization (NNMF)

- Let $\hat{M} = \text{Estimated number of received signals}$
- NNMF finds $\hat{A} \in \mathbb{R}_+^{T \times \hat{M}}$, $\hat{\Sigma} \in \mathbb{R}_+^{\hat{M} \times F}$ to minimize $\|E - \hat{A}\hat{\Sigma}\|_F$

Challenges:
- Estimating $\hat{M}$ is hard when $T < F$
  - Iteratively increase model size $\hat{M}$
- Non-convex cost function
  - Convergence to global minima not guaranteed
- Cost function is not probabilistic
  - Not robust to noise
- Non-unique solution and $\hat{A}$ is not binary
  - $\hat{\Sigma} \neq \Sigma$, i.e., thresholding $\hat{\Sigma}$ will not detect all occupied DFT bins

Our Proposed Solution

- Re-initialize multiple times
- Use energy detection to obtain binary time-freq. map
- Reconstruct each factor before detection

Reconstrcut each factor before detection
Proposed Algorithm: Overview

Initialization $\hat{M} = 1$

Energy Detection

Energy Detection

NNMF of $E'$ with $\hat{M}$ signals

Increment $\hat{M}$

No

Noise band detected?

Yes

Detect Occupied Bands

$\hat{B}_1, \hat{B}_2, ..., \hat{B}_{\hat{M}} \subseteq \{0, ..., F - 1\}$
Proposed Algorithm: Energy Detection

Initialization \( \hat{M} = 1 \)

Energy Detection

NNMF of \( E' \) with \( \hat{M} \) signals

Increment \( \hat{M} \)

\( E \)

\( E' \)

\( \hat{A}, \hat{\Sigma} \)

No

Noise band detected?

Yes

Detect Occupied Bands

\( \hat{B}_1, \hat{B}_2, \ldots, \hat{B}_{\hat{M}} \subseteq \{0, \ldots, F-1\} \)

\( \tau_v = \sigma_v^2 \left( 1 + \sqrt{2/NQ^{-1}(P_{fa})} \right) \)

Threshold [3]:

Proposed Algorithm: NNMF

Initialization \( \hat{M} = 1 \)

Energy Detection

Energy Detection

\( E \)

\( E' \)

NNMF of \( E' \) with \( \hat{M} \) signals

Increment \( \hat{M} \)

\( \hat{E} \)

\( \hat{A}, \hat{\Sigma} \)

No

Noise band detected?

Yes

Detect Occupied Bands

\( \hat{B}_1, \hat{B}_2, ..., \hat{B}_{\hat{M}} \subset \{0, ..., F - 1\} \)

Reconstructed Factors for \( \hat{M} = 4 \)

Signal energy shared by all factors

Significant signal energy

Noise Band
Proposed Algorithm: Detecting Bands

**Initialization**

\[ \hat{M} = 1 \]

**Energy Detection**

Energy Detection: \( E \)

\[ E' \]

**NNMF of** \( E' \) **with** \( \hat{M} \) **signals**

\[ \hat{A}, \hat{\Sigma} \]

**Increment** \( \hat{M} \)

**Noise band detected?**

- **No**
  - No
  - \( \hat{B}_1, \hat{B}_2, \ldots, \hat{B}_{\hat{M}} \subset \{0, \ldots, F - 1\} \)
  - **Detect Occupied Bands**

- **Yes**
  - \( \hat{\Sigma} \neq \Sigma \)
  - Signal energy shared by all factors
  - "Leaked" signal energy
  - Noise Band
Proposed Algorithm: Detecting Bands

Initialization $\hat{M} = 1$

1. Energy Detection $E$
2. $E'$
3. NNMF of $E'$ with $\hat{M}$ signals
4. $\hat{A}, \hat{\Sigma}$
5. Noise band detected? $\rightarrow$ Yes
6. Detect Occupied Bands

$\hat{B}_1, \hat{B}_2, ..., \hat{B}_M \subseteq \{0, ..., F - 1\}$

Challenge: $\hat{\Sigma} \neq \Sigma$ and unknown noise

Solution:
- Reconstruct and threshold peaks:
  \[
  \max_{t \in \{1, ..., T\}} \left( \hat{A}_m \hat{\Sigma}_m \right)_{tf} > 0.5
  \]

- **Active bin ignored if adjacent bins are not active**
  - Reduces false alarms

- **Ignore “duplicate” bands**
  - Similarity quantified by symmetric difference
Novel Performance Metrics: Why?

- Conventional wideband spectrum sensing metrics are per-bin
  - False alarm probability for each bin
  - Detection probability for each bin

Our Output

Ground Truth

Proposed Metrics:
- Number of detected bands
- Number of extra bands detected
- Relative Errors in Center Frequency and Bandwidth
Novel Performance Metrics: Why?

- Conventional wideband spectrum sensing metrics are per-bin
  - False alarm probability for each bin
  - Detection probability for each bin

Our Output

Ground Truth

Challenge
Match each detected band to the corresponding true band, if any
Novel Performance Metrics: How?

Our Output

\[
\hat{B}_1 \quad \hat{B}_2 \quad \hat{B}_3
\]

Ground Truth

\[
B_1 \quad B_2
\]

Edge Weights: 
\[
\delta \left( B_{m_1}, \hat{B}_{m_2} \right) \equiv F - \left| B_{m_1} \ominus \hat{B}_{m_2} \right|
\]

Symmetric Difference

Fully Connected Bipartite Graph
Novel Performance Metrics: How?

Our Output

Ground Truth

Solution: Find the Maximum Weight Matching

Edge Weights: \[ \delta\left( B_{m_1}, \hat{B}_{m_2} \right) \triangleq F - \left| B_{m_1} \ominus \hat{B}_{m_2} \right| \]

Symmetric Difference
Simulations: Performance vs. Activity

Receiver:
- Bandwidth 6MHz
- 512 length FFT, average of 100 windowed overlapping segments
- 25 measurements, i.e., ~1ms long

Transmitters:
- Bandwidth 600kHz each
- 4-PAM, pulse shaped signals
- Shadow fading channels with 6dB variance

Number of Detected Signals

Number of Extra Signals
Simulation: Performance vs. Spectral Overlap

Number of Detected Signals

Relative Error in Center Frequency

Number of Extra Signals

Relative Error in Bandwidth
Conclusions and Future Work

- Multiple power spectrum measurements can distinguish spectrally overlapped signals

- Conventional signal detection and estimation theory may not be sufficient

Future Work:

- Reduce number of extra signals detected
  - By improving non-negative matrix factorization methods?
- Estimate time of activity, i.e., $\hat{A}$, for use in traffic estimation
Thank you!

Questions?

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Selected References