Experimental Analysis of Cyclostationary Detectors Under Cyclic Frequency Offsets

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Abstract—Cyclostationary detection involves detecting cyclic features of modulated signals which are functions of various transmit parameters including the symbol rate, carrier frequency, and modulation format. However, imperfect knowledge of these transmit parameters at the sensing radio results in computing the detection test statistic under a cyclic frequency offset (CFO). The detection performance of the conventional cyclic autocorrelation function has been shown to degrade in the presence of the CFO impairment. In this paper, we propose a new multi-frame detection statistic that improves the robustness of the conventional cyclostationary detector to CFO impairments. The achievable gains in using this method over conventional detectors are quantified analytically and verified through hardware experiments using the Universal Software Radio Platform (USRP N200) transceivers.

I. INTRODUCTION

Motivated by the spectrum scarcity problem, Cognitive Radios (CRs) have been proposed to dynamically allocate unused spectrum to unlicensed users. In order to avoid interfering with licensed Primary Users (PUs), unlicensed Secondary Users (SUs) must sense the spectrum periodically for PU activity before accessing it. Thus, spectrum sensing is seen as a vital step needed for the effective operation of CRs. When the sensing is performed on a given time slot and a channel is determined to be free, CRs can then use the remaining of the time slot for transmission. Therefore, achieving reliable spectrum sensing within a low sensing time increases the overall throughput of the CR network.

Among many spectrum sensing techniques [1], cyclostationary detectors exploit the signal’s cyclostationarity [2], [3], which is a statistical property of all modulated signals. In addition, cyclostationary feature detectors exploit the stationarity property of the additive noise [4], which enables them to reliably detect signals even under the very low signal-to-noise (SNR) regime. Although one could theoretically suppress the noise at any SNR and ensure reliable signal detection given enough number of samples for averaging, this assumes that perfect knowledge of the cyclic frequency to be detected is available at the sensing radio.

The authors of [5] experimentally showed degradation of the cyclic features in the Spectral Correlation Function (SCF) in the presence of sampling clock offsets. Similarly, the authors of [6] studied the behavior of the cyclic features of modulated signals when imperfect knowledge of the transmitter’s cyclic frequency is assumed at the receiver. This is referred to as operating under a Cyclic Frequency Offset (CFO). This CFO can arise as a result of imperfect estimate of the transmit parameters, Doppler spread, or frequency synthesizer errors. It was shown in [6] that under a non-zero CFO, the cyclic feature used for detection decays with increasing sensing time, making reliable detection impossible at low SNRs. We refer to this behavior as a form of SNR wall, analogous in effect to the well known SNR wall issue [7] present in methods based on energy detection. Therefore, reliable signal detection cannot be achieved in the presence of CFOs below a certain SNR threshold, and as a result, cyclostationary detectors lose their advantage over energy detectors.

In this work, we present a new cyclic detector that achieves significant gains over conventional cyclic detectors in the presence of CFOs. We consider a wideband sensing radio sampling at a much higher rate than the bandwidth of each of the signals to be detected. The performance of the proposed detector is verified experimentally using the Universal Radio Software Platform USRP N200 transceivers with XCVR 2450 RF front-ends that operate in the ISM band. The results shown in this paper suggest that the proposed detector overcomes the SNR wall problem, and therefore can guarantee reliable signal detection at low SNRs.

The rest of this paper is organized as follows. Section II presents the system model and the problem statement. In Section III, we give an overview of the conventional cyclic detector, show its limitations in the presence of CFOs, and then present the proposed multi-frame cyclic detector. In Section IV, we optimize the detection performance of the proposed detector. Section V presents the experimental results that verify the achievable performance gains over conventional detectors, and finally Section VI concludes the paper.

II. SYSTEM MODEL

We focus on the detection of a single signal in a wideband channel in the presence of AWGN. The received signal can be expressed as follows

\[
x[n] = \begin{cases} 
  w[n] & \text{under } \mathcal{H}_0, \\
  \sum_m a(mT)p(nT_s - mT)e^{-j2\pi f_s nT_s} + w[n], & \text{otherwise}
\end{cases}
\]  

This work is supported by DARPA under grant A002069701.
where $T_s$ is the sampling period ($f_s = 1/T_s$ is the sampling frequency), $f_c$ is the carrier frequency, $\alpha(t)$ are the information symbols and $T$ is the symbol period. The wideband nature of the detection implies that $1/T \ll f_s$. Given a total of $N_T$ received samples, the objective is to detect the presence or absence of the signal of interest with a certain probability of detection and false alarm. The received signal is assumed to be a BPSK modulated signal which exhibit cyclic feature at cyclic frequency $\alpha = 1/T$. Note that the model applies to other linearly modulated signals as well. A list of the cyclic frequencies of other modulation classes can be found in [2] and the references within.

The goal of this work is to study the impact of imperfect knowledge of transmit parameters on the cyclic features used for signal detection.

### III. Performing Cyclostationary Detection

This section describes the conventional cyclic detector, and shows the challenges in spectrum sensing under low SNR regimes when a large number of samples is needed to perform reliable detection. We then present our proposed multi-frame cyclic detector and describe how it can enhance the detection performance.

#### A. Conventional Detector

Given that the signal is cyclostationary with cyclic frequency $\alpha$, the cyclic autocorrelation (CAC) function computes the correlation of the received signal with a delayed, frequency shifted, version of itself. We consider zero-lag CAC computations in this work. This can be performed as follows

$$R_x^*(\alpha) = \frac{1}{N_T} \sum_{n=0}^{N_T-1} x[n]x^*[n]e^{-j2\pi\alpha n T_s},$$  

(2)

where $N_T$ is the total number of samples. Under perfect knowledge of the signal’s symbol rate and carrier frequency, cyclostationary detectors can theoretically suppress noise at all SNRs with increasing sensing time by averaging of the stationary noise. As a result, cyclostationary detectors are seen as more reliable detectors at low SNRs since they do not suffer from the SNR wall phenomenon [7]. However, under imperfect knowledge of the transmit parameters, the test statistic is computed at a Cyclic Frequency Offset (CFO) from the true cyclic frequency. Under CFO $\Delta_\alpha$, the test statistic is computed at a new cyclic frequency,

$$\hat{\alpha} = \alpha(1 + \Delta_\alpha),$$

where $\Delta_\alpha$ can arise as a result of estimation errors, clock drifts, or Doppler spread in case of detection of a feature at the carrier frequency of the incoming signal. Since the noise is stationary, it is therefore unaffected by the CFO and therefore the null-hypothesis $H_0$ distribution is unaffected by this impairment. Since only the signal itself is affected by the CFO, we focus on studying the impact of the CFO on noiseless signals. It can be shown that the effect of the CFO on the conventional cyclic detector operating on noiseless signals is given by

$$|R_x^*(\hat{\alpha})| = |R_x^*(\alpha)| \frac{\sin(\pi\alpha N T_s)}{N T \sin(\pi\alpha N T_s)}, \quad (3)$$

Therefore, under a non-zero $\Delta_\alpha$, increasing the number of samples $N_T$ results in a decay of the cyclic feature. As a result, although increasing the sensing time averages out the noise, the cyclic feature used for detection gets attenuated as well, therefore degrading the detection performance. It is important to note that this effect cannot be compensated for with increasing sensing time. On the contrary, increasing the sensing time and therefore the total number of samples $N_T$ results in further degradation of the cyclic feature.

Therefore, conventional cyclic detectors fail at low SNRs in the presence of CFOs. This impairment can be seen as an SNR wall since increasing the sensing time does not improve the detection performance. The proposed detector is aimed at eliminating the SNR wall in the average sense, as is described in the next subsection.

#### B. Proposed Detector

Given that a large sensing time degrades the cyclic feature, we propose a multi-frame test statistic, where the total number of samples $N_T$ are split into $M$ frames of $N$ samples each. By shortening the number of samples per frame from $N_T$ to $N$, the effect of the CFO on the cyclic feature is less severe, while the remaining frames are used for statistical averaging of the noise. The proposed detector is a multi-frame cyclic detector given by

$$\hat{R}_x^*(\alpha) = \frac{1}{N_T} \sum_{k=1}^{M} \sum_{n=0}^{N-1} x_k[n]x^*_k[n]e^{-j2\pi\alpha n T_s}, \quad (4)$$

where $k$ is the frame index, $M$ is the number of frames, $N$ is the number of samples per frame such that $NM \leq N_T$. By limiting the number of samples per frame to $N$ samples, the degradation factor is therefore less severe. However, due to the nature of frame-based processing, there will be a phase offset from one frame to the next whenever $N\alpha T_s$ is non-integer. Intuitively, whenever the statistic does not go through an integer number of cycles of its cyclic frequency within $N$ samples, there will be a phase offset with respect to the next frame. Note that if this phase offset is compensated for in each frame, the result becomes equivalent to the single-frame test statistic, reducing the proposed statistic to the conventional one. It turns out that the effect of CFOs on the proposed cyclic detector under noiseless conditions is given by

$$|\hat{R}_x^*(\hat{\alpha})| = |\hat{R}_x^*(\alpha)| \frac{\sin(\pi\alpha N T_s) \sin(\pi\alpha N MT_s)}{N^2 \sin^2(\pi\alpha N T_s)} \frac{M \sin(\pi\alpha N T_s)}{N \sin(\pi\alpha N T_s)} \quad (5)$$

As expected, when $N = N_T$ and $M = 1$, the multi-frame test statistic reduces to the conventional cyclic detector (3). In addition, the highest gain that can be achieved using the multi-frame test statistic is whenever $N$ can be chosen such that $N\alpha T_s$ is an integer. Under such scenario, the maximum
gain for a given number of samples per frame, $G(N)$, is given by
\[
G(N) = \left| \frac{N_T \sin(\pi \alpha N T_s) \sin(\pi \alpha N T_s)}{N \sin(\pi \alpha N T_s) \sin(\pi \alpha N T_s)} \right|. \tag{6}
\]

However, the maximum gain cannot always be achieved since the relationship between the sampling rate and the cyclic frequency can be such that no feasible $N$ can make $N \alpha T_s$ an integer. Under such a case, performance suffers due to non-coherent integration of the frames. Although resampling can be performed in order to make $N \alpha T_s$ an integer, this solution suffers from two drawbacks: 1) the resampling has to be performed for every signal to be detected in the wideband spectrum, which either requires additional sensing delay or architectural complexity, and 2) the cyclic frequency is not perfectly known at the sensing radio, therefore making $N \alpha T_s$ an integer is not realizable.

With respect to the noise hypothesis, it can be shown that the distribution of the test statistic under $H_0$ does not change under the multi-frame test statistic for the same number of samples $N_T$. Therefore, we focus on the effect of the multi-frame test statistic (4) on the signal only case. In the next section, we show how the cyclic feature of the signal can be maximized in the average sense, where the averaging is performed over the statistics of the CFO.

IV. OPTIMIZATION PROBLEM FORMULATION

If the CFO $\Delta_\alpha$ was known, one could compensate for it by changing the cyclic frequency at which the test statistic is being computed. However, since the CFO arises as a result of an uncertainty in the cyclic frequency, it can be modeled as a random variable and therefore only a statistical description of this impairment can be obtained. We assume prior knowledge of the probability distribution function $p_\Delta(\cdot)$ of the CFO $\Delta_\alpha$, which could be obtained experimentally in practice.

The optimization is aimed to maximize the average detection performance of the signal of interest. As a result, our goal is to maximize the average cyclic feature of the noiseless-signal, where the average is taken with respect to the distribution of the CFO. From (5), the factor that governs the decay of the cyclic feature is a function of the CFO $\Delta_\alpha$ and, the number of samples per frame $N$, and the number of frames $M$. As such, the only design variable under our control is the way the total number of samples $N_T$ are split into $M$ frames of $N$ samples each. Therefore, the goal is to select the best $(N, M)$ pair that maximizes the average cyclic feature.

Given a $(N, M)$ pair such that $NM \leq N_T$, we compute the average decay $F_{(N, M)}(\Delta_\alpha)$ of the cyclic feature
\[
F_{(N, M)}(\Delta_\alpha) = \int_{-\infty}^{\infty} \left| \frac{\sin(\pi \alpha N T_s) \sin(\pi \alpha N T_s)}{N \sin(\pi \alpha N T_s) \sin(\pi \alpha N T_s)} \right| p_\Delta(\Delta_\alpha) d\Delta_\alpha. \tag{7}
\]

For simplicity, we assume that the CFO is uniformly distributed over $[0, \Delta_\alpha]$, however the analysis holds for any other distribution as well. Therefore, (7) simplifies to
\[
F_{(N, M)}(\Delta_\alpha) = \int_{0}^{\infty} \left| \frac{\sin(\pi \alpha N T_s) \sin(\pi \alpha N T_s)}{N \sin(\pi \alpha N T_s) \sin(\pi \alpha N T_s)} \right| d\Delta_\alpha. \tag{8}
\]

Note that in order to capture the cyclostationary property of the signal being processed, a minimum number of symbols need to be spanned within each frame for the test statistic to converge. Therefore, based on the sampling rate and the cyclic frequency of the signal, there exists a minimum number of samples per frame $N_{min}$ that needs to be spanned. The optimization formulation that maximizes the average cyclic feature can therefore be written as
\[
(\hat{N}, \hat{M}) = \arg \max_{N, M} F_{(N, M)}(\Delta_\alpha)
\]

such that $NM \leq N_T, N \geq N_{min} \tag{9}$

Note that the optimum pair $(\hat{N}, \hat{M})$ that maximizes the average cyclic feature might be such that the total number of samples is less than the total acquired samples $N_T$. The block diagram of the proposed system is shown in Fig. 1.

V. EXPERIMENTAL RESULTS

In this section, we describe our experimental setup used to verify the performance of both the single-frame and multi-frame test statistics as a function of total number of samples and CFO.

To conduct hardware experiments we used the USRP N200 transceivers with XCVR 2450 RF front-ends that can operate in the ISM band. The USRP N200 is a software radio that allows for high-speed streaming capability up to 50 MS/s in both directions (8-bit samples). When coupled with the XCVR 2450 radio front-ends, the USRP allows transmission at 2.4 GHz or 5.9 GHz range. We implemented a BPSK baseband modulator followed by an up-converter emulating a PU in the ISM band which is connected to a sensing radio (SU) via an SMA cable. This setup eliminates any undesired channel impairments such as having multipath components. The transmitter is modulating a BPSK signal with a symbol period $T = 10 \mu s$ which is transmitted via an SMA cable. At the receiver end, the incoming signal is attenuated using a 30 dB attenuator. The receiver USRP is controlled by its own computer where we have implemented both single and multi-frame cyclic detectors. The cyclic frequency at which the cyclic detector computes the statistic is modeled as a random variable between 0 and 1000 ppm.

The transmit symbol rate is set to 100 KHz and the receive sampling rate is set at either $f_s = 2$ MHz ($N \alpha T_s$ integer can be satisfied for any $N$), or $f_s = 2.1739$ MHz ($N \alpha T_s$ integer cannot be satisfied). The transmit signal is centered at $f_c = 5.16$ GHz, and the transmit power is set such that the average receive SNR is 20 dB in order to study the effect of CFO on pseudo-noiseless signals. On the receiver end, the sensing radio computes both the conventional and multi-frame test statistics at $\alpha = 1/T = 100$ KHz which would be compared to a decision threshold to determine the presence...
In the first experiment, we compute the conventional test statistic (3) for different number of total samples \( N_T \) at fixed CFOs \( \Delta_\alpha \) of 500, 1000, and 3000 parts per million (ppm), which was controlled by computing the test statistic on the receiver side at an offset from the true cyclic frequency. Fig. 2 shows that increasing the sensing time and therefore the total number of samples, the cyclic feature of interest degrades under non-zero CFO. As a result, when large number of samples are needed at low SNR to suppress the noise, the cyclic feature decays as well creating an SNR wall where reliable signal detection is impossible. It is clear that the rate of decay of the cyclic feature gets worse with higher CFOs. This experiment verifies the behavior of the cyclic features using the conventional cyclic detector, and motivates the need for a solution to this problem.

Next, we analyze the behavior of the cyclic feature of the signal of interest using the proposed cyclic detector when the receive sampling rate has been set to \( f_s = 2 \) MHz which yields an integer \( N\alpha T_s \) value. This is the case where the maximum gain can be achieved with the proposed detector because of the absence of the phase offset from one frame to the next. Fig. 3 shows the degradation of the cyclic feature under a fixed CFO of 1000 ppm versus the number of samples per frame for different number of frames \( M = 1, 2 \) and 5. As is illustrated, splitting the total number of samples into frames decreases the rate at which the cyclic feature decays, therefore making cyclic detection using the proposed detector more reliable. Again, the theoretical curve matches with the experimental data, verifying that this is the maximum possible achievable gain using the proposed method. However, this gain cannot always be achieved in wideband channels when \( N\alpha T_s \) cannot be guaranteed to be an integer. This case is studied in the next experiment.

In this experiment, we tackle the more general case of wideband spectrum sensing where the number of samples per frame cannot be chosen such that \( N\alpha T_s \). This can occur for two reasons: 1) the number of samples required to make \( N\alpha T_s \)
Fig. 4. Cyclic feature as a function of cyclic frequency offsets under $N_T = 51000$ samples, and $M = 1, 2, 3, 4$ and 5 frames.

Fig. 5. Average cyclic feature with $\Delta _\alpha = 1000$ppm for both the conventional and proposed cyclostationary detectors.

an integer might be too large, 2) the receiver does not have perfect knowledge of the cyclic frequency $\alpha$ which makes it impossible to set the frame length accordingly. The sampling rate is set to $f_s = 2.173913$ MHz on the sensing radio, and the cyclic feature to be detected is kept at $\alpha = 100$ KHz. The aim of this experiment is to show the degradation to the cyclic feature when coherent integration of the frames is not possible. Fig. 4 shows the rate of decay of the cyclic feature versus the cyclic frequency offsets for a fixed number of total samples $N_T = 51000$. The incoming samples are split into frames of $M = 1, 2, 3, 4$, and 5 frames, and the cyclic feature is plotted under each of these 5 cases versus the CFO impairment, which is limited between 0 and 1000 ppm. As derived theoretically, non-coherent integration will result in a decaying factor given by

$$\frac{\sin(\pi \alpha NMT_s)}{M \sin(\pi \alpha NT_s)}$$

(10)

As shown in Fig. 4, the penalty factor (10) due to the multi-frame processing varies as a function of the number of frames $M$. Another way of looking at this result is that the choice of the way the total number of samples are split determines the rate at which the feature degrades. Again, experimental data matches closely with the theoretical predicted behavior.

In the next experiment, we quantify the achievable gains achieved by the proposed detector over conventional cyclic detectors. For a range of sensing times, we solve the optimization problem given in (9) numerically with $\Delta _\alpha = 1000$ ppm, and obtain the optimum way to split the acquired $N_T$ samples. Fig. 5 shows the comparison of the average cyclic feature with both the conventional and the proposed multi-frame test statistic. As expected, the single-frame conventional average test statistic decays gradually with increasing number of total samples, making the detection impossible at low SNRs. However, using the multi-frame cyclic detector, the average cyclic feature is shown to be robust to the CFO impairment. In fact, although the optimum split is to use the single cyclic detector for some $N_T$, a slight increase in the sensing time can result in large gains in the average cyclic feature. As a result, this experiment verifies that the proposed detector does not suffer from the SNR wall phenomenon making this detector robust to the CFO impairment.

VI. CONCLUSION

We have shown in this paper the effect of cyclic frequency offsets on the conventional cyclic detector which results in a degradation of the cyclic feature used for signal detection. We have proposed a new multi-frame cyclic detector that is more robust to the cyclic frequency impairment. Experimental data from USRP N200 transceivers have been used to verify the validity of the proposed method in different scenarios. Our results show that although conventional detectors fail in low SNR regimes in the presence of cyclic frequency offsets, the proposed detector can still detect the cyclic features reliably, therefore overcoming the SNR wall phenomenon.

REFERENCES