A Modulation Dependent Channel Coherence Metric for VANET simulation using IEEE 802.11p

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Abstract—The most common physical layer models for network simulations are the bit error rate and the SNR threshold model. In time-varying channels such as those experienced in vehicular networks, these models are assumed valid as long as the packet duration is less than the coherence time of the channel. The coherence time is a statistical measure of the channel invariance. It is independent of the signal parameters or the receiver structure. While it is convenient to decouple the system performance from the channel, this paper shows that this simplification may lead to inaccurate performance assessments and erroneous conclusions. This paper suggests a new metric, the normalized empirical coherence time (NETC), based on results from an extensive simulation campaign of a typical IEEE 802.11p system. The NETC delineates the minimum time (as a percentage of signal duration) over which the system achieves some performance threshold. The metric is explicitly a function of modulation, packet duration, and the traditional coherence time. This new metric could be used in place of the traditional coherence time as a constraint on the packet duration necessary to assume channel variation has negligible impact on performance.

I. INTRODUCTION

Vehicular networks exist in inherently time-varying environments. For accurate simulation the variation of the channel with time must be taken into account. Traditionally, coherence time has been used for this purpose. Typical IEEE 802.11p receivers do not track the channel in time [1]. Rather, it is assumed that as long as the packet duration is less than the coherence time of the channel, the average error rate performance will be dictated by the instantaneous signal-to-noise ratio (SNR). This basic performance can be found analytically or through simulations over a typical, time-invariant channel. Common physical layer (PHY) models used in network simulation compute packet errors based on the resulting average bit error rate (BER) curves or an SNR threshold set according to the target error rate (essentially a binary partitioning of the BER curve).

While it is convenient to assume that a packet shorter than the coherence time of a channel experiences no degradation due to the channel time-variation, this paper shows this to be false. It is also false to assume that a packet longer than the coherence time will be severely degraded. The severity of degradation depends on the signal and receiver structures as well. In time-varying channels, the channel estimates derived from the initial preamble lose coherence with the channel over the duration of the packet. Thus, longer packets experience greater error rates due to accumulated channel estimation error [2]. This applies even to the slow-fading scenario, which is the focus of this paper, where the channel is essentially invariant over the symbol duration and there is negligible ICI.

Several papers have addressed error rate performance of uncoded narrowband communications over quasi-static channels accounting for imperfect channel state information (CSI) for both single-input single-output [3], [4] and multiple-input multiple-output [5], [6] systems. Analysis in [7] applied to coded modulation in correlated fading but with perfect CSI. A narrowband bit-interleaved, coded modulation system with imperfect CSI was analyzed in [8] for 16QAM symbols only. In all of these analyses the error rate was not related to either the packet length or coherence time of the channel. In most cases, the channel was considered quasi-static, varying independently from symbol to symbol. None of the analyses applied directly to a practical system or standard. The authors of [2], [9] investigated the transmission efficiency of IEEE 802.11R/A, a short-lived precursor to the current IEEE 802.11p standard, over time-varying channels. Using simulated packet error rate (PER) versus velocity curves, they computed the ratio of the ideal transmission time of a packet to that including physical and link layer overhead.

This paper defines and quantifies a new metric, the normalized empirical coherence time (NETC). The NETC provides a more accurate estimate of the time over which a system may be considered unaffected by the time-varying channel. The metric demarcates the greatest time, normalized by packet duration, such that the system performance is degraded by some percentage. It is an explicit function of modulation. By dint of its derivation from empirical PER curves, similar to [2], the metric implicitly accounts for the signal, receiver, and channel structures which determine the underlying PER. These results have particular relevance to the simulation of VANETs which tend to operate over time varying channels and whose appropriate packet lengths are unknown.
II. SYSTEM MODEL

A baseband equivalent IEEE 802.11 OFDM model was used for an extensive simulation campaign. We assume that the received signal distortion is solely from the doubly-selective channel. Thus we assume ideal time and frequency synchronization. The receiver generated channel estimates from the long training sequence (LTS) of the packet preamble using a zero-forcing estimator. The frequency domain observations, along with the channel estimates, were input to a mismatched detector which generated approximate log-likelihood ratios subsequently input to a hard-decision Viterbi decoder. Embedded pilots were not used for channel tracking. While the model was sub-optimal compared to maximum-likelihood detection, it was representative of typical practical implementations.

A. System Parameters

A number of system parameters were swept across a range of values summarized in Table I. By setting high SNR, any observed performance degradation would be dominated by channel estimation error and not noise. Packet lengths are set in terms of OFDM symbols to fix the packet duration across modulations. The packet length in bytes is proportional to the channel bandwidth. The receiver generated channel estimates from the long training sequence (LTS) of the packet preamble using a zero-forcing estimator. The frequency domain observations, along with the channel estimates, were input to a mismatched detector which generated approximate log-likelihood ratios subsequently input to a hard-decision Viterbi decoder. Embedded pilots were not used for channel tracking. While the model was sub-optimal compared to maximum-likelihood detection, it was representative of typical practical implementations.

B. Channel Parameters

The channel model was a wide-sense stationary, uncorrelated scattering model. Several channels were used in the simulations: a set of canonical channel models (CC) and an Expressway channel model (GEC) derived from a channel sounding campaign by researchers at the Georgia Institute of Technology ([12, Annex Q], [13]). The CC have conventional, though synthetic scattering functions whereas the GEC models a realistic channel. The CC scattering function is exponential in delay and uniform in Doppler. That is, the gain $g_l$ and Doppler spread $f_{D_l}$ of tap $l = 1..L$ with delay $\tau_l$ are

$$g_l = C e^{-a\tau_l} \quad \text{and} \quad f_{D_l} = f_D,$$

respectively. The decay rate $a$ of the tap gains is set to engender a particular coherence bandwidth [14]

$$B_C = 0.2/\tau_{rms},$$

where $\tau_{rms} = \sqrt{\sum_{l=1}^{L} g_l^2}$ and $\tau_l < T_{CP}$ the cyclic prefix duration. Equivalently $L < BT_{CP}$ where $B=10$ MHz is the channel bandwidth. The normalizing constant $C$ ensured that the channel had unit gain (i.e. $C^{-1} = \sum_{l=1}^{L} e^{-a\tau_l}$). The gains were constrained such that $10\log_{10}(g_1/g_L) \leq 30$ dB. Recall for the CC $g_1 > g_2 > \ldots > g_L$.

The uniform Doppler spread $f_D$ was set to engender a particular coherence time $T_C$ as defined by [14]

$$T_C \triangleq 0.423/f_D$$

where $f_D = 2\pi \sqrt{\int f^2 S(f) df}$ and $S(f)$ is the (normalized) power spectral density of the Doppler spectrum. For the classic Jakes spectrum used for the CC, $f_D = \sqrt{2\pi f_m}$ where $f_m$ is the maximum Doppler shift. The Doppler spectra for the GEC are not the Jakes spectrum, however their Doppler spreads are upper bounded by those of the Jakes spectrum. Table II summarizes the channel parameters for both the CC and GEC.

Because the CC were intended to act as a proxy for V2V channels, a line-of-sight component was assumed. The first tap Rice K-factor is given as $k_1$. The CC were not meant to emulate a real channel but to serve as references for channels with particular coherence time and coherence bandwidth measures. The coherence times are chosen to be comparable to the packet durations such that the coherence time to packet duration ratio is in the neighborhood of one.\(^1\)

\(^1\)For curious readers, the CC coherence times correspond to mobile speeds of approximately 34 to 675 mph (55 to 1086 km/h) at an operating frequency of 5.9 GHz.

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### Table I

**Parameter Settings For Simulation Campaign**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>30 dB</td>
</tr>
<tr>
<td>Packet length</td>
<td>2 – 30 OFDM symbols</td>
</tr>
<tr>
<td>Modulation</td>
<td>BPSK, QPSK, 16QAM, 64QAM</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>2.5, 5, 10, 20 MHz</td>
</tr>
</tbody>
</table>

### Table II

**Channel Parameters For Simulation Campaign**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CC</th>
<th>GEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>3–15</td>
<td>11</td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>300–1500 ns</td>
<td>450 ns</td>
</tr>
<tr>
<td>$\tau_{rms}$</td>
<td>26.6–200 ns</td>
<td>48 ns</td>
</tr>
<tr>
<td>$B_C$</td>
<td>1–7.5 MHz</td>
<td>4.2 MHz</td>
</tr>
<tr>
<td>$f_D$</td>
<td>1.3–26.4 kHz</td>
<td>0.813 kHz</td>
</tr>
<tr>
<td>$T_C$</td>
<td>16–320 $\mu$s</td>
<td>520 $\mu$s</td>
</tr>
<tr>
<td>$k_1$</td>
<td>0 dB</td>
<td>21.9 dB</td>
</tr>
<tr>
<td>$g_1/g_L$</td>
<td>30 dB</td>
<td>28 dB</td>
</tr>
<tr>
<td>$S(f)$</td>
<td>Jakes</td>
<td>varied*</td>
</tr>
</tbody>
</table>

* see [12, Annex Q]
III. METHODOLOGY

For the vehicular scenario motivating our research, safety applications are a particular priority. These applications have strict latency and reliability requirements. Our performance metric is point-to-point link layer throughput. This metric is inversely proportional to latency. Thus a maximum latency constraint would correspond to a minimum throughput, given some message size (in bytes). Throughput also gives an upper bound on the number of users supported by a particular link. For example, if the supported link throughput is 1 Mbps and the tolerable latency is 20 ms for a 500 bit message, then $10^6/500 \times 0.02 = 40$ users could be supported with perfect scheduling.

At a high level, PER curves are obtained via simulation of a typical IEEE 802.11p system for a range of modulations, packet lengths, signal bandwidths, and channel coherence times. The PER curves are transformed to throughput via calculations detailed in the sequel. Trends in the throughput versus coherence time, packet duration ratio are fit to an analytical expression. The expression is then used to define the NETC.

A. Packet Error Rate

Simulations were run for a minimum of 10000 packets or 400 packet errors. We were interested in degradation due solely to accumulated channel estimation error. Because the channel estimation algorithm uses the preamble LTS, as long as $f_D \ll f_\Delta$, where $f_\Delta = B/64 \approx 156$ kHz is the sub-carrier separation, the frequency-domain estimates were not significantly degraded by ICI. In order to ensure that the channel impulse response remained within the duration of the cyclic prefix to avoid inter-symbol interference (ISI) regardless of the signal bandwidth, analysis in this paper utilizes results for the shortest channel (equivalently the largest $B_C=7.5$ MHz) only. Error performance over channels with smaller $B_C$ was actually upper-bounded by the performance at the largest $B_C$. In other words, the results presented can be interpreted as worst case results (of the channels simulated). Because the standard has channel coding across sub-carriers, increased frequency-domain diversity results in greater coding gain. Unless otherwise stated signal bandwidth is 10 MHz.

Fig. 1 shows the packet error rate versus packet length for each modulation and several coherence times. For any particular coherence time in Fig. 1 the QPSK and 16QAM curves outperform those for BPSK and 64QAM. This trend suggests a tradeoff between modulation order (hence, constellation density or minimum inter-symbol distance) and packet duration. While high order modulations result in shorter packets and thus accumulate less channel estimation error, their dense constellations are more susceptible to that error. Conversely, low order modulations result in long packets with greater accumulated error, but less sensitivity due to their large minimum symbol distance. The QPSK and 16QAM packets offer the best tradeoff (of the simulated modulations) between packet duration and estimation error sensitivity.

\begin{align*}
  T_L &\triangleq N(L_o + L_p)T_s \\
  N &= \arg\min_N F_X(D; p_c, K) \geq 1 - P_M \\
  D &= N - K, \quad K = M/L_pRT_s
\end{align*}

where $N$ is the total number of packets transmitted (including those dropped and re-transmitted $D$) and $T_s$ is the OFDM symbol period (including the cyclic prefix). $K$ is the number of packets required to complete a $M$ bit message sent at PHY rate $R$ bps using packets consisting of $L_p$ OFDM symbols each. In words, $N$ is the minimum number of packets transmitted to ensure $K$ successful packets are received with probability $1 - P_M$. By computing $N$ according to $P_M$ we explicitly tie the acceptable performance to the reliability constraints. Network analyses like [15], [16] account for network contention but compute only average throughput independent of any constraints.

Random variable $X$ is negative-binomial distributed with cumulative distribution function (CDF) $F_X(\cdot)$ and probability of (packet) error $p_c$. The negative-binomial distribution gives the probability of $D$ failures before $K$ successes over a series
of Bernoulli trials given success rate $q = 1 - p_e$. The CDF is

$$F_X(D; q, K) = I_q(K, D + 1) = \frac{B(q; K, D + 1)}{B(K, D + 1)}$$

where $I_x(a, b)$ is the regularized incomplete beta function. $B(x; a, b)$ (8) is the integral form of the incomplete beta function and $B(a, b) = B(1; a, b)$.

For this analysis, the message error rate $P_M = 0.1$ and the message size $M = 8000$ bits. Fig. 2 plots the results of applying (4) and (5) to the curves of Fig. 1.

C. Throughput

In the final transformation, we invert the link layer latency to obtain an estimate of the link layer throughput

$$R_{LL} \triangleq \frac{M}{T_L}.$$  

Fig. 3 shows the latency curves of Fig. 2 transformed according to (9). Particularly for short coherence times, Fig. 3 shows that there is an optimal packet length selection that maximizes throughput. It also shows that the optimal packet length is dependent on the modulation.

By substituting (4) and (6) into (9), we can express the throughput as

$$R_{LL} = \frac{M}{N(L_p + L_o)T_s} = \frac{R K L_p}{N L_p + L_o} = \frac{R \rho}{1 + \epsilon} \approx R \rho (1 - \epsilon).$$  

(10)

where $\rho \triangleq K/N$ is the degradation due to re-transmissions and $\epsilon \triangleq L_o/L_p$ is the degradation due to per packet overhead. The final approximation of (10) results from the first-order approximation of $1/(1 + \epsilon)$. Because PER increases with packet length, $\epsilon$ and $\rho$ are inversely related. That is, as packet length increases $\epsilon$ decreases (naturally) while $\rho$ increases. The optimal packet length provides the best tradeoff between the two sources of degradation.

To concisely display results across the range of bandwidths, modulations, packet lengths, and coherence times Fig. 4 shows a scatter plot of the empirical bandwidth efficiency ($\beta_e$) versus normalized coherence time ($\bar{T}_C$) where

$$\beta_e \triangleq R\rho/B = \beta \frac{\rho}{T_s}.$$  

(11)

The empirical bandwidth efficiency is the traditional bandwidth efficiency degraded by the overhead of re-transmissions. To isolate this overhead from per-packet overhead the empirical throughput was multiplied by $1/(1 + \epsilon)$ which is equivalent to setting $\epsilon = 0$ in (10).

The plot illustrates trends dependent on modulation and $\bar{T}_C$ only. The dashed lines labeled “trad $T_C$” show the efficiency according to the traditional coherence time, explicitly $\beta I(\bar{T}_C > 1)$ where $I(x)$ is the indicator function. The trend lines of Fig. 4 are the least-square error (LSE) fit to the data of the function

$$f(x) = \frac{v_1}{x + e^{-v_2 x}} + v_3$$  

(13)

where $\vec{v} = \{v_1 \ldots v_4\}$ were determined via an optimization procedure. The function in (13) is a generalization of the hyperbolic tangent function, which results by setting $\vec{v} = \{2, 2, -1, 2\}$, chosen for its characteristic shape.
D. NETC

Two measures can be computed from the trend line equations. The plateau is the bandwidth efficiency achieved for large $\hat{T}_C$. Equal to $v_3 + v_4$, the plateau measures the maximum achievable performance, i.e. the performance over an effectively static channel. In this region only per-packet overhead $\epsilon$ degrades performance. The corner is defined as the normalized coherence time at which we achieve 90% of the plateau performance. The corner is an estimate of the normalized empirical coherence time (NETC). The NETC demarcates the minimum coherence time to packet duration ratio above which the channel impact on performance is considered negligible. Table III summarizes the optimized parameter sets for the trend lines of Fig. 4.

Analytically, the plateau should occur at $\lim_{v \to 1} \beta = \beta$, the underlying PHY efficiencies 0.3, 0.6, 1.2, and 2.4 bps/Hz for BPSK, QPSK, 16QAM, and 64QAM modulations respectively. The plateaus in Table III generally agree with the expected values except for 64QAM. The plateau is underestimated due to the uniform error weighting across the domain and the paucity of points at high $T_C$ (in Fig. 4 the rightmost data points for 64QAM achieve 2.4 bps/Hz). This would also result in an underestimate of the NETC. The NETC (corner in Table III) varies significantly with modulation. For BPSK and QPSK one can even achieve acceptable performance with a packet duration greater than the traditional channel coherence time. A similar metric based on the traditional channel coherence time would correspond to 1 for all modulations.

Table III summarizes the optimized parameter sets for the trend lines of Fig. 4.

![Table III: Trend Line Parameters, Plateau, and Corner](image)

<table>
<thead>
<tr>
<th>Modulation</th>
<th>$v_1$</th>
<th>$v_2$</th>
<th>$v_3$</th>
<th>$v_4$</th>
<th>plateau</th>
<th>corner</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>1.44</td>
<td>4.56</td>
<td>-0.48</td>
<td>0.78</td>
<td>0.29</td>
<td>0.57</td>
</tr>
<tr>
<td>QPSK</td>
<td>1.14</td>
<td>3.19</td>
<td>-0.62</td>
<td>1.21</td>
<td>0.59</td>
<td>0.95</td>
</tr>
<tr>
<td>16QAM</td>
<td>1.87</td>
<td>1.75</td>
<td>-1.13</td>
<td>2.31</td>
<td>1.18</td>
<td>1.80</td>
</tr>
<tr>
<td>64QAM</td>
<td>1.41</td>
<td>0.76</td>
<td>-1.11</td>
<td>3.30</td>
<td>2.20</td>
<td>4.59</td>
</tr>
</tbody>
</table>

IV. DISCUSSION

Equation (13) and Table III were used to create a parametric approximation for Fig. 3. The “fit” curves provide a smooth inter/extrapolation of the simulation results while providing a means of generating similar throughput versus packet length curves for a larger set of channels (not simulated). Fig. 5 shows the synthesized curves along with simulation results extended to large packets for $CC T_C = 320 \mu s$, $B_C = 7.5$ MHz. For comparison, a similar transformation was performed using the “trad TC” function of Fig. 4. The circles mark the packet lengths corresponding to the traditional coherence time. The squares mark the maximum packet length according to the NETC. The “fit” curves capture the major features of the simulated behavior while the “trad TC” curves diverge for longer packet lengths. As mentioned in the previous paragraph, the NETC for 64QAM is underestimated. For each of the other modulations, the NETC packet length is slightly greater than that corresponding to the peak throughput. This is a consequence of our allowance of 10% degradation in our determination of the NETC.

These parametrized curves facilitate generation of results for a wider range of $T_C$ with greater resolution. Since (13) is a function of normalized, not absolute, coherence time, the NETC is applicable to the entire class of channels with the scattering function detailed in II-B, but with a scaled Doppler spread. This may be particularly applicable to the vehicular scenario where the scattering function is highly dependent on the geography while the Doppler spread is a function of the speed of traffic.

To ascertain the applicability of the canonical channel results to non-canonical channels, Fig. 6 plots throughput versus packet length from simulations of the GEC and parametrized curves using (13). For all curves $T_C = 65$ symbols. The parametrized curves only approximate the optimal packet length. They also provide a loose upper bound on the achievable throughput at higher packet lengths. These inaccuracies would suggest that the fine structure of the channel scattering function has an impact on the actual performance beyond their effect on the coherence time and coherence bandwidth measures.

Despite the error between the “fit” curves and the simulation results, the packet lengths corresponding to the NETC (squares) provides a better bound with which to guarantee throughput performance than does the traditional coherence time (circles). Table IV shows the maximum packet length using the NETC and the throughput maximizing packet length from simulation for two signal bandwidths. The NETC provides a much better guide than does the traditional coherence time of 65 symbols.

V. CONCLUSIONS

This paper defined and derived the normalized empirical coherence time which indicates the maximum coherence time (normalized to packet length) necessary to ensure negligible degradation of the link layer throughput of a typical IEEE 802.11p system operating over a time-varying channel. Given a fixed channel with a particular coherence time, the NETC

![Fig. 5. Throughput versus packet length for $T_C=320 \mu s$.](image)
... was shown to provide a more accurate bound, compared to the traditional coherence time, on the maximum packet length resulting in negligible throughput degradation. While the metric is based on a particular packet, receiver, and channel structure, the NETC provides better guidance than the traditional coherence time in general channels.

For network simulations assuming a non-tracking IEEE 802.11p receiver, the NETC should be used to guide the maximum packet length for which only per-packet overhead impacts throughput. In other words, simulations that assume increasing packet length decreases overhead and thus increases throughput should limit their maximum packet length according to the NETC to maintain valid assumptions. This could have particular impact on network aggregation or coding schemes that rely on large packets.

Future work will quantify those channel parameters that most influence upper layer performance. The impact of alternative signal and receiver structures will also be explored. The goal is to link the signal, channel, and receiver parameter sets to upper layer performance. Such a relationship could provide much needed detail to VANET simulations for network-wide performance evaluation.

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**REFERENCES**


