Millimeter-wave technologies offer several advantages over lower frequency bands, including:

- **More capacity**
- **Peak rate**
- **Level latency**
- **Long battery lifetime**
\[
H(t, f) = \frac{1}{\sqrt{N_t N_r}} \sum_{l=1}^{L} \sum_{k=1}^{K_l} g_{l,k} a_r(\phi_{l,k}) a_t^H(\theta_{l,k}) e^{j2\pi f \tau_{l,k}} \delta(t - \tau_{l,k})
\]

\[
H = \frac{1}{\sqrt{N_t N_r}} \sum_{l=1}^{L} g_{l} a_r(\phi_{l}) a_t^H(\theta_{l})
\]

\[
a_r(\phi) = [1, e^{j\pi \sin(\phi)}, \ldots, e^{j\pi N_t \sin(\phi)}]^T,
\]

\[
a_t(\theta) = [1, e^{j\pi \sin(\theta)}, \ldots, e^{j\pi N_r \sin(\theta)}]^T
\]
1. Directional beam pattern (heuristic) [4]
1. Directional beam with width adaptation [5]
   - Not supporting multi-UE IA; requires feedback
1. -
   -
   -
   -
   -
IA procedure

Within slot $m$, $n \in m = \{n: (m-1)N \leq n \leq mN\}$

- TX: transmit pilot sequence using precoding weight $w[n] = w_m$
- Rx: receive using combining weight $v[n] = v_m$

\[
y[n] = v[n]^H H w[n - d] s[n - d] e^{j(2\pi \Delta f T_s n + \Psi[n])} + z[n]
\]

- To design & Known to system
- To estimate

\[
y[n] = \Delta f, d, H
\]

<table>
<thead>
<tr>
<th>$M$</th>
<th>Num. of beam pattern pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_m, v_m$</td>
<td>Beamformers in slot $m$</td>
</tr>
<tr>
<td>$s[n]$</td>
<td>Pilots</td>
</tr>
<tr>
<td>$d, \Delta f$</td>
<td>Timing &amp; Freq. offset</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Sampling interval</td>
</tr>
<tr>
<td>$\Psi[n]$</td>
<td>Phase noise process</td>
</tr>
<tr>
<td>$z[n]$</td>
<td>AWGN</td>
</tr>
</tbody>
</table>
\[ y[n] = \sum_{m=1}^{M} v_m^H H w_m s_m[n] e^{j(2\pi \Delta f T_s n + \Psi[n])} + z[n] \]

- Known to system
- To estimate

\( s_m[n] \) Pilots (finite support in \( m \))

1. \( \Delta f \)
2. \( h_m \)
3. \( H \)
4. \( y[n] \)
\[
\angle y[n] - \angle y[n-1] = \tan^{-1} \frac{\Re(y[n])\Im(y[n-1]) - \Re(y[n-1])\Im(y[n])}{\Re(y[n])\Re(y[n-1]) + \Im(y[n])\Im(y[n-1])}
\]

\[
\Delta \hat{f} = \frac{\angle y[n] - \angle y[n-1]}{2\pi T_s}
\]

\[
\begin{bmatrix}
\angle y[n] \\
\Delta f
\end{bmatrix}
= \begin{bmatrix}
1 & 2\pi T_s \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\angle y[n-1] \\
\Delta f
\end{bmatrix}
+ \begin{bmatrix}
\Delta \Psi[n] \\
0
\end{bmatrix}
+ \begin{bmatrix}
\cos(x_1) \\
\sin(x_1)
\end{bmatrix} + v_n
\]

\[
z_n = \begin{bmatrix}
\Re(y[n]) \\
\Im(y[n]) \\
\Delta \hat{f}
\end{bmatrix}^T
\]
- **Prediction Steps**
  \[ \hat{x}_{n|n-1} = F \hat{x}_{n-1|n-1} \]
  \[ P_{n|n-1} = FP_{n-1|n-1}F^T + Q \]

- **Updating Steps**
  \[ \tilde{y}_n = z_n - h(\hat{x}_{n|n-1}) \]
  \[ S_n = H_n P_{n|n-1} H_n^T + R \]
  \[ K_n = P_{n|n-1} H_n^T S_n^{-1} \]
  \[ \hat{x}_{n|n} = \hat{x}_{n|n-1} + K_n \tilde{y}_n \]
  \[ P_{n|n} = (I_2 - K_n H_n) P_{n|n-1} \]

\[ Q = \text{diag} \left( \sigma_{\Delta \Psi}^2, 0 \right) \]

\[ \mathbf{R} \text{ represents on-shot error, a function} \]

\[ \mathbf{H}_n \triangleq \frac{\partial z_n}{\partial \mathbf{x}_n} \]
\[ = \begin{bmatrix} -\sin(\{\hat{x}_{n|n-1}\}_1) & -\cos(\{\hat{x}_{n|n-1}\}_1) \\ 0 & 0 \end{bmatrix} \]

\[ h_m \quad y[n] \]

\[ h_m \quad h_{m-1} \]
\[
\begin{align*}
    h_m &= v_m^H A_r \tilde{G} A_t^H w_m \\
    H &= c_i a_r \left( \frac{\pi}{6} \right) a_i^H \left( \frac{\pi}{4} \right) \\
    \tilde{G} &= \mathbb{C}^{N_r \times N_t} \\
    L &= \mathbb{C} \\
    g &= \text{vec} \left( \tilde{G} \right)
\end{align*}
\]
The proposed initial BF training performance is nearly same as compressive channel sounding with perfect synchronization.
There is 10X spectrum efficiency degradation in compressive BF training with phase errors.

The proposed algorithm reaches high beam alignment with various phase error setting.