MBS-SPS
Signal processing for the 60 GHz band

by Johan van den Heuvel

Traineeship Report
carried out from October 2005 to December 2005.

Supervisors:
Dr. R. Rietman (Philips)
Dr. ir. P.C.W. Sommen (TU/e)

The Faculty of Electrical Engineering of Eindhoven University of Technology disclaims all responsibility for the contents of traineeship and graduation reports.
Abstract

The research in this report is done in the framework of the 60 GHz band research at the Philips NatLab Connectivity Systems and Networks Group. The 60 GHz band is of great interest since there is a massive spectral space (5 GHz) allocated worldwide for dense wireless communication. To use the spectral space a reliable and low-cost interconnection is needed. Because the 60 GHz band is primarily used for indoor applications, the received power of reflections bouncing of walls and floors can be larger than the received power of the direct line of sight signal. To enhance the received power from the direct line of sight signal and to suppress most reflections smart antenna structures are needed to optimize signal transmission and reliability. Important parts of the smart antenna structure are the signal processing and the antenna element. Due to the large bandwidth of the signal the power consumption of the AD-converters, needed in smart antenna structures, becomes too large to have one AD-converter per antenna. Therefore a part of the signal processing needs to be done in the analog domain. The smart antenna structure is used for beam forming. The ability of the antenna structure to "beam" is limited by the antenna element which is used. Due to production-cost and chip-integration considerations, the antenna focused on is the rectangular patch antenna. To model the communication between the transmitting antenna and the receiving antenna-array a channel model is introduced. In this model every receiving antenna has a direct line of sight signal and a reflected signal from a surface, where the reflected signal is considered to be smaller or equal to the direct line of sight signal. According to theory and simulations the signal to noise ratio of an antenna array which is matched for the input signal can be matched or bettered by an array twice the size consisting out of a multiplier and a delay. Taking into account that a simpler filter design reduces significantly the complexity of the algorithm needed to optimize the signal to noise ratio, since the filters can be optimized separately, it is preferred over a more complex solution. It also became clear that the main effect of the patch antenna element is a limitation to the angles at which the antenna array can "beam". The preferred antenna element is a square patch antenna. The position of the feed point changes the ability of the array to beam in certain angles. An antenna array consisting out of differently orientated patches could be considered to enhance the ability to "beam" in more angles. An other alternative is a patch array where the feed point of the patch can be chosen.
## Contents

1 Introduction .............................................................. 1

2 Filter design ............................................................ 3

   2.1 Introduction ....................................................... 3

   2.2 Filter design for one receiver ................................. 4

      2.2.1 Introduction ................................................. 4

      2.2.2 Matched filter .............................................. 4

      2.2.3 Optimizing a filter ......................................... 6

      2.2.4 Conclusions .................................................. 8

   2.3 Filter-array design for receiver-array structures .......... 8

      2.3.1 Introduction ................................................. 8

      2.3.2 Optimizing filter-arrays for receiver-array structures . 8

      2.3.3 Conclusions .................................................. 11

   2.4 Channel model ..................................................... 11

      2.4.1 Introduction ................................................. 11

      2.4.2 One transmitter and a receiver array .................... 11

      2.4.3 Conclusions .................................................. 13

   2.5 Simulating filter designs for one receiver ................. 13

      2.5.1 Introduction ................................................. 13
3.5 Simulating isotropic antenna arrays . . . . . . . . . . . . . . . . . . 44
  3.5.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . 44
  3.5.2 Radiation pattern . . . . . . . . . . . . . . . . . . . . . . . 44
  3.5.3 Conclusions . . . . . . . . . . . . . . . . . . . . . . . . . . 46
3.6 Simulating a patch antenna . . . . . . . . . . . . . . . . . . . . . . . 50
  3.6.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . 50
  3.6.2 Radiation pattern . . . . . . . . . . . . . . . . . . . . . . . 50
  3.6.3 Conclusions . . . . . . . . . . . . . . . . . . . . . . . . . . 52
3.7 Simulating patch antenna arrays . . . . . . . . . . . . . . . . . . . 52
  3.7.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . 52
  3.7.2 Radiation pattern . . . . . . . . . . . . . . . . . . . . . . . 52
  3.7.3 Conclusions . . . . . . . . . . . . . . . . . . . . . . . . . . 57
3.8 Conclusions . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 57
4 Conclusions and recommendations . . . . . . . . . . . . . . . . . . . . 59
Chapter 1

Introduction

The research in this report is done in the framework of the 60 GHz band research at the Philips NatLab Connectivity Systems and Networks Group. The 60 GHz band is of great interest since there is a massive spectral space (5 GHz) allocated worldwide for dense wireless communication [1]. To use the spectral space a reliable and low-cost interconnection is needed. Because the 60 GHz band is primarily used for indoor applications, the received power of reflections bouncing of walls and floors can be larger than the received power of the direct line of sight signal [2]. To enhance the received power from the direct line of sight signal and to suppress most reflections smart antenna structures are needed to optimize signal transmission and reliability [3, 4, 5, 6, 2]. Important parts of the smart antenna structure are the signal processing and the antenna element.

Due to the large bandwidth of the signal the power consumption of the AD-converters, needed in smart antenna structures, becomes too large to have one AD-converter per antenna. Therefore a part of the signal processing needs to be done in the analog domain. The smart antenna structure is used for beam forming [6] [7]. The ability of the antenna structure to ”beam” is limited by the antenna element which is used. Due to production-cost and chip-integration considerations, the antenna focused on is the rectangular patch antenna [8, 9, 10, 11, 12, 13].

This report focusses on the signal to noise ratio which can be attained by simple filter structures in the analog domain and on the limitations imposed by antenna elements on beam forming. In Chapter 2 an introduction into filter design is given and simulations are done. In Chapter 3 a model is derived to calculate the radiation pattern of a patch antenna array and simulations are done. Finally Chapter 4 gives the conclusions and recommendations of the report.
Chapter 2

Filter design

2.1 Introduction

Because there is a massive spectral space allocated ($5 \text{GHz}$) at the $60 \text{GHz}$ band the power consumption of the AD-converters is such that having one AD-converter per receiving antenna would be too power consuming. To save on AD-converters a part of the signal processing needs to be done in the analog domain. This Chapter focusses on the signal to noise ratio (SNR) which theoretically can be attained before the signal is sampled by an AD-converter. The channel model which is introduced in Section (2.4) assumes one direct line of sight signal and one reflection per receiving antenna (Figure (2.1)). It is also assumed every receiving antenna has its own filter and the output of every filter is combined before being sampled by an AD-converter. Therefore the analog filters need to be characterized as such that the total array has the maximal SNR before sampling. The analog components of the filters are controlled by a digital processor which is behind the AD-converter. To get an insight into the behavior of a filter an analytical model will be derived in Section (2.2) to characterize a given filter as such that the signal to noise ratio is maximal. In Section (2.3) the theory will be expanded to include filter-array structures. In Section (2.4) the used channel model is presented. Simulations of various filter designs are done in Section (2.5) and Section (2.6). The simulations are done to get an insight in the filter structures which are needed in the analog domain. The preferred structure should be as simple as possible, since this reduces the number of variables which need to be set for an optimal SNR by the digital processor.
Figure 2.1: Channel model for one transmitting antenna and a receiving antenna array.

2.2 Filter design for one receiver

2.2.1 Introduction

The elementary case of one receiver is introduced to give an insight in filter matching techniques. First the matched filter is introduced, since the matched filter is the filter which will yield the best SNR possible for a given signal [14]. Due to cost considerations or for practical reasons it is not always possible to build an analog matched filter. Therefore the theory will be expanded as such that a given filter design can be characterized to give the maximal SNR possible for that filter.

2.2.2 Matched filter

To maximize signal to noise ratio the matched filter is used [14]. The input signal is defined as $s(t)$ and the output signal as $s_0(t)$. The input noise is defined as $n(t)$ and the output noise as $n_0(t)$. The matched filter is used in applications where the signal may or may not be present, but when the signal is present its wave-shape is known. The signal is assumed to be time limited on the interval $(0, T)$ and is zero otherwise. The power spectral density $P_n(f)$, of the additive input noise $n(t)$ is also known. The filter characteristics need to be determined such that the instantaneous output signal power is maximized at a sampling time $t_0$ compared to the average output noise power. Therefore the filter $F(f)$
is defined so that

\[
\left( \frac{S}{N} \right)_{\text{out}} = \frac{s_0^2(t)}{n_0^2(t)}. \tag{2.1}
\]

has a maximum at \( t = t_0 \). This is the matched filter criterion.

The output signal at time \( t_0 \) is

\[
s_0(t_0) = \int_{-\infty}^{\infty} F(f)S(f)e^{i2\pi f t_0} df \tag{2.2}
\]

and the average power of the noise is

\[
n_0^2(t) = R_{n0}(0) = \int_{-\infty}^{\infty} |F(f)|^2 P_n(f) df. \tag{2.3}
\]

Substituting these two equations into Equation (2.1) gives

\[
\left( \frac{S}{N} \right)_{\text{out}} = \frac{\left| \int_{-\infty}^{\infty} F(f)S(f)e^{i2\pi f t_0} df \right|^2}{\int_{-\infty}^{\infty} |F(f)|^2 P_n(f) df} \tag{2.4}
\]

According to the matched filter criterion the filter \( F(f) \) which maximizes Equation (2.4) needs to be found. The filter \( F(f) \) can be found by using the Schwarz inequality:

\[
\left| \int_{-\infty}^{\infty} A(f)B(f) df \right|^2 \leq \int_{-\infty}^{\infty} |A(f)|^2 df \int_{-\infty}^{\infty} |B(f)|^2 df \tag{2.5}
\]

where \( A(f) \) and \( B(f) \) may be complex functions of the real variable \( f \). Equality is obtained only when

\[
A(f) = K_c B^*(f) \tag{2.6}
\]

where \( K_c \) is an arbitrary constant. The Schwarz inequality is used to replace the numerator on the right hand side of Equation (2.4) by letting

\[
A(f) = F(f)\sqrt{P_n(f)} \tag{2.7}
\]

and

\[
B(f) = S(f)e^{i2\pi f t_0} \sqrt{P_n(f)}. \tag{2.8}
\]
Then Equation (2.4) becomes

\[
\frac{S}{N}_{\text{out}} \leq \frac{\int_{-\infty}^{\infty} |F(f)|^2 P_n(f)df \int_{-\infty}^{\infty} |S(f)|^2 df}{\int_{-\infty}^{\infty} |F(f)|^2 P_n(f)df} \tag{2.9}
\]

where \(P_n(f)\) is a nonnegative real function. Thus,

\[
\frac{S}{N}_{\text{out}} \leq \int_{-\infty}^{\infty} \frac{|S(f)|^2 df}{P_n(f)} \tag{2.10}
\]

The maximum signal to noise ratio is obtained when the filter \(F(f)\) is chosen such that equality is attained (Equation (2.6)). This occurs when,

\[
F(f) = K_c \frac{S^*(f)e^{-i2\pi f t_0}}{P_n(f)} \tag{2.11}
\]

### 2.2.3 Optimizing a filter

Due to cost considerations or for practical reasons it is not always possible to construct an analog matched filter. Therefore the theory is expanded such that a given filter can be characterized to maximize the signal to noise ratio at a given sampling time \(t_0\). One way to characterize a given filter to maximize the SNR at the sampling time \(t_0\) is to calculate when the derivative of every variable is equal to zero. The problem with this technique is that there is no guarantee that the result is not a sub maximum of the function. This problem was avoided in the previous section by using the Schwarz inequality. If it is therefore possible to use the Schwarz inequality it would be clear the maximal SNR is found. From theory it is known any given function can be described by an infinite sum of orthogonal functions (the equality of Parseval) \[15] [16]. If the noise spectral power density function is also introduced into the orthogonal functions it is possible to use the Schwarz inequality and the maximal SNR can be found. Since the orthogonal functions can be chosen freely it is possible to introduce the noise spectral power density function inside the integral equations which define the orthogonal space. If this technique is applied to rewrite the variables of the filter as a sum of orthogonal functions the filter \(F(f)\) is defined as:

\[
F(f) = \langle \sum_{n=1}^{N} a_n e_n(f) \rangle e^{-i2\pi f t_0}, \tag{2.12}
\]

where \(a_n\) are complex constants and \(e_n\) an orthogonal space defined as
\[ \int_{-\infty}^{\infty} |e_n(f)|^2 P_n(f) df = 1, \]
\[ \int_{-\infty}^{\infty} e_n^*(f)e_m(f)P_n(f) df = 0, \]
\[ \int_{-\infty}^{\infty} e_n(f)e_{m*}(f)P_n(f) df = 0, \]  
(2.13)

where \( m \neq n \) and \( P_n(f) \) is the spectral power density of the noise and \( N \) is the number of orthogonal functions. Now Equation (2.4) can be written as

\[
\left( \frac{S}{N} \right)_{\text{out}} = \frac{| \int_{-\infty}^{\infty} \sum_{n=1}^{N} a_n e_n(f)S(f) df |^2}{\int_{-\infty}^{\infty} | \sum_{n=1}^{N} a_n e_n(f) |^2 P_n(f) df},
\]
(2.14)

where \( S(f) \) is the input signal and \( t_0 \) the sampling time. Because of the definition of \( e_n \) this equation can be written as

\[
\left( \frac{S}{N} \right)_{\text{out}} = \frac{| \sum_{n=1}^{N} a_n h_n |^2}{\sum_{n=1}^{N} |a_n|^2 \int_{-\infty}^{\infty} |e_n(f)|^2 P_n(f) df}.
\]
(2.15)

where

\[
h_n = \int_{-\infty}^{\infty} e_n(f)S(f) df.
\]
(2.16)

By using the Schwarz inequality (Equation (2.5)) the equation can be written as

\[
\left( \frac{S}{N} \right)_{\text{out}} \leq \frac{\sum_{n=1}^{N} |a_n|^2 |h_n|^2}{\sum_{n=1}^{N} |a_n|^2 \int_{-\infty}^{\infty} |e_n(f)|^2 P_n(f) df}.
\]
(2.17)

Which equals

\[
\left( \frac{S}{N} \right)_{\text{out}} \leq \sum_{n=1}^{N} |h_n|^2.
\]
(2.18)

The maximum signal to noise ratio is obtained when \( a_n \) is chosen such that equality is attained in Equation (2.18). This occurs only when the complex constants \( a_n \) are chosen such that

7
\[ a_n = K_c h_n^*. \] (2.19)

where \( K_c \) is any arbitrary constant. If the found complex constants \( a_n \) and the defined functions \( e_n(f) \) are used in Equation (2.12) the optimized filter in respect to the SNR \( F(f)_{\text{optimal}} \) is found

\[ F(f)_{\text{optimal}} = K_c (\sum_{n=1}^{N} h_n^* e_n(f)) e^{-i2\pi ft_0}, \] (2.20)

where \( K_c \) is any arbitrary constant. The signal to noise ratio of the output of this optimized filter is found with Equation (2.18) and is given by

\[ \left( \frac{S}{N} \right)_{\text{out}}^{\text{max}} = \sum_{n=1}^{N} |h_n|^2. \] (2.21)

### 2.2.4 Conclusions

By using the Schwarz inequality an analytical model is derived to characterize any filter structure as such that the signal to noise ratio is maximal. In the next Section the theory will be expanded to include array structures.

### 2.3 Filter-array design for receiver-array structures

#### 2.3.1 Introduction

Since smart antenna structures are necessary on the 60GHz band the theory of the previous section will be expanded to include array structures. Therefore an analytical model will be derived to characterize a given filter-array structure as such that the signal to noise ratio is maximal in respect to a given channel model, transmitter-array structure and receiver-array structure.

#### 2.3.2 Optimizing filter-arrays for receiver-array structures

To maximize the signal to noise ratio for a given receiver- and filter-array, where every receiver has its own distinct filter and all filter outputs are combined (Figure (2.1)), the theory is expanded such that a given filter-array can be characterized to maximize the signal to noise ratio at a given sample time \( t_0 \).
The signal of the transmitter-array is defined as a vector $S(f)$ of size $M$ and the channel model as a matrix $H(f)$ of size $M \times K$. The receiver-array and thus the filter-array is defined as a vector $F(f)$ of size $K$. Where $F(f)$ is defined so that for a given filter array $F(f)$

\[
\begin{align*}
\left( \frac{S}{N} \right)_{out} &= \frac{s_0^2(t)}{n_0^2(t)}
\end{align*}
\]

has a maximum at $t = t_0$. To find the optimal values in respect to the SNR of the variables of vector $F(f)$ the derivatives of every variable need to be set to zero. The problem with this method is that there is no guarantee the found solution is not a sub maximum of the function. This problem was avoided in the previous section by using the Schwartz inequality. If it is therefore possible to rewrite the vector $F(f)$ as such that the Schwartz inequality can be used the maximal SNR can be found. From theory it is known any given function can be described by an infinite sum of orthogonal functions (the equality of Parseval) [15][16]. If the noise spectral power density function is also introduced into the orthogonal functions it is possible to use the Schwartz inequality and the maximal SNR can be found. Since the orthogonal functions can be chosen freely it is possible to introduce the noise spectral power density function inside the integral equations which define the orthogonal space. If this technique is applied to rewrite the variables of the elements of the filter array as a sum of orthogonal functions the elements of vector $F(f)$ are defined as:

\[
F_k(f) = \left( \sum_{n=1}^{N} a_{kn} e_{kn}(f) \right) e^{-i2\pi ft_0}
\]

where $F_k(f)$ is the $k^{th}$ element of vector $F(f)$, $a_{kn}$ are complex constants and $e_{kn}$ an orthogonal space defined as

\[
\begin{align*}
\int_{-\infty}^{\infty} |e_{kn}(f)|^2 P_n(f) df &= 1 \\
\int_{-\infty}^{\infty} e_{kn}^*(f) e_{km}(f) P_n(f) df &= 0 \\
\int_{-\infty}^{\infty} e_{kn}(f) e_{kn}^*(f) P_n(f) df &= 0
\end{align*}
\]

where $m \neq n$ and $P_n(f)$ is the spectral power density of the noise and $N$ is the number of orthogonal functions. Now Equation (2.4) can be written as

\[
\left( \frac{S}{N} \right)_{out} = \frac{\int_{-\infty}^{\infty} F(f) H(f) S(f) e^{i2\pi ft_0} df}{\int_{-\infty}^{\infty} |F(f)|^2 P_n(f) df},
\]

(2.25)
which equals

\[
\left( \frac{S}{N} \right)_{out} = \left| \int_{-\infty}^{\infty} \sum_{k=1}^{K} \sum_{n=1}^{N} a_{kn} e_{kn}(f) H(f) S(f) df \right|^2 \left( \sum_{k=1}^{K} \sum_{n=1}^{N} a_{kn} e_{kn}(f) \right)^2 P_n(f) df. \tag{2.26}
\]

Because of the definition of \( e_{kn} \) this equation can be written as

\[
\left( \frac{S}{N} \right)_{out} = \left| \sum_{k=1}^{K} \sum_{n=1}^{N} a_{kn} h_{kn} \right|^2 \left( \sum_{k=1}^{K} \sum_{n=1}^{N} |a_{kn}|^2 \int_{-\infty}^{\infty} |e_{kn}(f)|^2 P_n(f) df \right)^2. \tag{2.27}
\]

where

\[
h_{kn} = \int_{-\infty}^{\infty} e_{kn}(f) H(f) S(f) df. \tag{2.28}
\]

By using the \textit{Schwarz inequality} (Equation (2.5)) the equation can be written as

\[
\left( \frac{S}{N} \right)_{out} \leq \left( \sum_{k=1}^{K} \sum_{n=1}^{N} |a_{kn}|^2 \right) \left( \int_{-\infty}^{\infty} |e_{kn}(f)|^2 P_n(f) df \right)^2. \tag{2.29}
\]

Which equals

\[
\left( \frac{S}{N} \right)_{out} \leq \sum_{k=1}^{K} \sum_{n=1}^{N} |h_{kn}|^2. \tag{2.30}
\]

The maximum signal to noise ratio is obtained when \( a_{kn} \) is chosen such that equality is attained. This occurs only when the complex constants \( a_{kn} \) are chosen such that

\[
a_{kn} = K_c h_{kn}^*. \tag{2.31}
\]

where \( K_c \) is any arbitrary constant. If the found complex constants \( a_{kn} \) and the defined functions \( e_{kn}(f) \) are used in Equation (2.23) the optimized elements of the filter in respect to the SNR \( F_k(f)_{\text{optimal}} \) are found

\[
F_k(f)_{\text{optimal}} = K_c \left( \sum_{n=1}^{N} h_{kn}^* e_{kn}(f) \right) e^{-i2\pi ft_0}, \tag{2.32}
\]


where $K_c$ is any arbitrary constant. The signal to noise ratio of the output of this optimized filter array is found with Equation (2.30) and is given by

$$\left( \frac{S}{N} \right)_{out}^{max} = \sum_{k=1}^{K} \sum_{n=1}^{N} |h_{kn}|^2. \tag{2.33}$$

2.3.3 Conclusions

By using the Schwarz inequality an analytical model is derived to characterize any filter-array structure as such that the signal to noise ratio is maximal in respect to a given channel model and transmitter-array.

2.4 Channel model

2.4.1 Introduction

Since at the 60 GHz band smart antenna structures are needed a model needs to be introduced to describe the communication between the transmitting antenna and the receiver antenna-array.

2.4.2 One transmitter and a receiver array

To model the communication between the transmitting antenna and the receiving antenna-array a channel model is introduced (Figure (2.2)). In this model every receiving antenna has a direct line of sight signal and a reflected signal from a surface. The signal at the $k^{th}$ receiving antennas is

$$S(f)_{antenna\, k} = S(f)_{transmitter}((1 + D_k e^{-i2\pi f T_{dk}}) e^{-i2\pi f R_k/c}) \tag{2.34}$$

where $S(f)_{transmitter}$ is the signal of the transmitting antenna, $D_k$ is the attenuation of the reflected signal, $R_k$ is the distance between the receiver and the transmitter, $T_{dk}$ is the delay time of the reflected signal in respect to the direct line of sight signal and $c$ is the speed of light. The time delay $T_{dk} = p_k/c$, where $p_k$ is the extra path length of the reflected signal compared to the direct line of sight signal. In this chapter $S(f)_{transmitter}$ will be a flat-band signal for the regarded bandwidth.
Figure 2.2: Channel model for one transmitting antenna and a receiving antenna array.

\[
S(f)_{\text{transmitter}} = \begin{cases} 
    s_1, & (f_c - \frac{B}{2}) \leq f \leq (f_c + \frac{B}{2}) \\
    0, & f \text{ otherwise}
\end{cases} \quad (2.35)
\]

where \( s_1 \) is a complex constant, \( B \) is the bandwidth and \( f_c \) the center frequency of the signal. In this case the model of the channel at the \( k \)th antenna will be

\[
S(f)_{\text{antenna}_k} = s_1 ((1 + D_k e^{-i2\pi f T_{dk}}) e^{-i2\pi f R_k / c}). \quad (2.36)
\]

The matched filter for the \( k \)th antenna is found with help of Equation (2.10) and Equation (2.11)

\[
F(f)_{\text{matched}_k} = \frac{K_c}{P_n(f)} s_1 ((1 + D_k e^{-i2\pi f T_{dk}}) e^{-i2\pi f R_k / c}) e^{-i2\pi f t_0}. \quad (2.37)
\]

where \( K_c \) is an arbitrary constant. The noise is considered to be white noise [14]. The power spectral density in the case of white noise is

\[
P_n(f) = \frac{1}{2} N_0 \quad (2.38)
\]

where
\[ N_0 = kT \] (2.39)

with \( k \) Boltzmann’s constant \( (k = 1.38 \times 10^{-23} \text{ joule/K}) \) and \( T \) the temperature \( (T = 290 \text{ K}) \). In this case the signal to noise ratio of the \( k^{th} \) matched filter will be

\[
\left( \frac{S}{N} \right)_{out,k} = \frac{|s_1|^2}{2N_0} \left( B + \frac{D_k}{\pi T_{dk}} \left( \sin(2\pi T_{dk}(f_c + B/2)) - \sin(2\pi T_{dk}(f_c - B/2)) \right) + BD_k^2 \right)
\] (2.40)

The total SNR can be derived with help of Equation (2.30)

\[
\left( \frac{S}{N} \right)_{out} = \sum_{k=1}^{K} \frac{|s_1|^2}{2N_0} \left( B + \frac{D_k}{\pi T_{dk}} \left( \sin(2\pi T_{dk}(f_c + B/2)) - \sin(2\pi T_{dk}(f_c - B/2)) \right) + BD_k^2 \right)
\] (2.41)

where \( K \) is the total number of the receiving antennas.

### 2.4.3 Conclusions

A model to describe the communication between the transmitter and the receiver antenna-array has been presented. The model will be used in the next sections for simulations.

### 2.5 Simulating filter designs for one receiver

#### 2.5.1 Introduction

To get an insight in the filters which are needed to obtain sufficient SNR, first the elementary case of one transmitter and one receiver is considered. To reduce the number of variables which need to be set by the digital processor, which is behind the AD-converter, the filter structures need to be as simple as possible. Therefore structures which have the least number of variables are preferred. To compare the different filters the difference in signal to noise ratio of the output of the optimized considered filter in respect to a matched filter is used. The used input signal is a flat-band signal with one reflection, as was presented in the previous section.
2.5.2 One amplifier

First the simplest filter structure is considered. This structure is a filter consisting of only one amplifier

\[ F(f) = c_1 e^{-i2\pi ft_0} \]  \hspace{1cm} (2.42)

where \( c_1 \) is a complex constant and \( t_0 \) the sampling time. To find the optimal value of \( c_1 \) in respect to the signal to noise ratio the theory of the previous section is used and the filter is described as a sum of subfunctions. Now the filter can be rewritten as

\[ F(f) = \left( \sum_{n=1}^{N} a_n e_n(f) \right)e^{-i2\pi ft_0} \]  \hspace{1cm} (2.43)

where \( e_n(f) \) is defined according to Equation (2.13). For this case the orthogonal space becomes

\[ e_1 = \frac{1}{\sqrt{B}} \frac{1}{\sqrt{\frac{1}{2}N_0}} = b_1. \]  \hspace{1cm} (2.44)

The signal at the receiving antennas is

\[ S(f)_{\text{antenna}} = S(f)_{\text{transmitter}} (1 + De^{-i2\pi fT_d}) \]  \hspace{1cm} (2.45)

where \( S(f)_{\text{transmitter}} \) is the signal of the transmitting antenna, \( D \) is the attenuation of the reflected signal, \( T_d \) is the delay time of the reflected signal in respect to the direct line of sight signal and \( c \) is the speed of light. The time delay \( T_d = p/c \), where \( p \) is the extra path length of the reflected signal compared to the direct line of sight signal. The distance between the transmitter and the receiver is not taken into account, because only one receiver antenna is considered. In this chapter \( S(f)_{\text{transmitter}} \) will be a flat-band signal for the regarded bandwidth.

\[ S(f)_{\text{transmitter}} = \begin{cases} s_1, & (f_c - \frac{B}{2}) \leq f \leq (f_c + \frac{B}{2}) \\ 0, & f \text{ otherwise} \end{cases} \]  \hspace{1cm} (2.46)

where \( s_1 \) is a complex constant, \( B \) is the bandwidth and \( f_c \) the center frequency of the signal. To find the maximal value of the SNR first the value of \( h_n \) needs to be calculated. The value of \( h_n \) can be derived by using Equation (2.16) and is equal to
\[ h_1 = B b_1 s_1 - \frac{b_1 D s_1}{i 2 \pi T_d} \left( e^{-i 2\pi fT_d(f_c + B/2)} - e^{-i 2\pi fT_d(f_c - B/2)} \right). \tag{2.47} \]

Now the maximal signal to noise ratio (SNR) is found with Equation (2.18) and equals

\[ \left( \frac{S}{N} \right)_{\text{out}}^{\text{max}} = |h_1|^2. \tag{2.48} \]

The result is compared to the signal to noise ratio (SNR) of a matched filter for different bandwidths (Figure(2.3)). The difference in SNR of the matched filter and the used filter is

\[ \Delta \text{SNR} = \text{SNR}_{\text{filter}} - \text{SNR}_{\text{matched filter}}. \tag{2.49} \]

where the SNR is defined as

\[ \text{SNR} = 10 \times \log_{10} \left( \left( \frac{S}{N} \right)_{\text{out}} \right). \tag{2.50} \]

For simulations the noise was considered to be white noise, the attenuation \( D = \frac{1}{4} \), the path-length \( p = 1 \) m and the center frequency \( f_c = 60 \) GHz. Figure (2.3) shows that the \( \Delta \text{SNR} \) stabilizes for larger values of the bandwidth \( B \). This can be derived from Equation (2.49)

\[ \Delta \text{SNR} = 10 \times \log_{10} \left( \frac{\left( \frac{S}{N} \right)_{\text{out filter}}}{\left( \frac{S}{N} \right)_{\text{out matched filter}}} \right). \tag{2.51} \]

where

\[ \left( \frac{S}{N} \right)_{\text{out filter}} = \frac{|s_1|^2}{\frac{D}{2 \pi T_d B} \left( \sin(2\pi T_d(f_c + B/2)) - \sin(2\pi T_d(f_c - B/2)) \right) + \frac{D(2 - 2 \cos(2\pi T_d B))}{B^2 (2\pi T_d)^2}}. \]

\[ \left( \frac{S}{N} \right)_{\text{out matched filter}} = \frac{|s_1|^2}{\frac{D}{2 \pi T_d B} \left( \sin(2\pi T_d(f_c + B/2)) - \sin(2\pi T_d(f_c - B/2)) \right) + D^2}. \tag{2.52} \]

If the bandwidth \( B \) goes to infinity \( \Delta \text{SNR} \) becomes,
Figure 2.3: $\Delta$SNR as a function of bandwidth for $D = \frac{1}{4}$.

\[
\lim_{B \to \infty} \Delta \text{SNR} = 10 \times \log_{10} \left( \frac{1}{1 + D^2} \right). \tag{2.53}
\]

Since the attenuation $D$ varies between $0 \leq D \leq 1$ it is shown with Equation (2.53) that for the worst case, where the attenuation $D = 1$, the $\Delta$SNR has a value of $-3 \text{dB}$ for large bandwidths.

### 2.5.3 Two amplifiers and a small fixed delay

To improve the $\Delta$SNR for the worst case a more complex filter is considered. The considered filter is a filter with two amplifiers and a delay. To reduce the number of variables the delay is set to be constant. The output of a filter with a delay is written in the time domain as

\[
f(t)_{\text{out}} = d_1 s(t) + d_2 s(t - \delta) \tag{2.54}
\]

where $\delta$ is a delay, $d_1$ and $d_2$ are complex constants and $s(t)$ is the input signal. The filter $f(t)$ and $F(f)$ are linked according to the Fourier transform

\[
F(f) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi ft} dt
\]

\[
f(t) = \int_{-\infty}^{\infty} F(f) e^{j2\pi ft} df \tag{2.55}
\]
To find the value for the delay the equation is rewritten with

\[ d_1 = c_1 - \frac{c_3}{\delta} \]
\[ d_2 = \frac{c_3}{\delta} \]  \hspace{1cm} \text{(2.56)}

now \( f(t)_{\text{out}} \) becomes

\[ f(t)_{\text{out}} = c_1 s(t) + c_3 \frac{s(t) - s(t - \delta)}{\delta} \]  \hspace{1cm} \text{(2.57)}

The second part of the found equation looks similar to the definition of a derivative. According to the definition of a derivative \( \delta \) should go to zero. If \( \delta \) goes to zero the equation equals:

\[ \lim_{\delta \to 0} f(t)_{\text{out}} = c_1 s(t) + c_3 \frac{ds(t)}{dt} \]  \hspace{1cm} \text{(2.58)}

The Fourier transform of \( f(t) \) for small delays, \( F(f) \) is

\[ F(f) = c_1 + c_2 f \]  \hspace{1cm} \text{(2.59)}

taking into account the sampling time \( t_0 \) the equation becomes

\[ F(f) = (c_1 + c_2 f)e^{-i2\pi ft_0} \]  \hspace{1cm} \text{(2.60)}

where \( c_1 \) and \( c_2 = j2\pi c_3 \) are complex constants and \( f \) is the frequency. To find the optimal values of \( c_1 \) and \( c_2 \) in respect to the SNR the filter is defined as in Equation (2.43). The orthogonal space for this case is

\[ e_1 = \frac{1}{\sqrt{B}} \frac{1}{\sqrt{\frac{1}{2}N_0}} = b_1 \]
\[ e_2 = \frac{1}{\sqrt{\pi B^2}} (f - f_c) \frac{1}{\sqrt{\frac{1}{2}N_0}} = b_2 (f - f_c) \]  \hspace{1cm} \text{(2.61)}

To find the maximal value of the SNR first the values of \( h_n \) need to be calculated. The values of \( h_n \) can be derived by using Equation (2.16) and are equal to

\[ h_1 = Bb_1 s_1 - \frac{b_1 Ds_1}{i2\pi T_d} (e^{-i2\pi fT_d(f_0+B/2)} - e^{-i2\pi fT_d(f_0-B/2)}) \]  \hspace{1cm} \text{(2.62)}
The maximal value of the SNR is found with Equation (2.18) and is equal to

$$\left( \frac{S}{N} \right)_{\text{max}}^{\text{out}} = |h_1|^2 + |h_2|^2. \quad (2.64)$$

The found maximal SNR for this filter is compared to the matched filter and the filter with only one amplifier. Results are shown in Figure (2.4). For simulations the noise was considered to be white noise, the attenuation $D = \frac{1}{4}$, the path-length $p = 1 \text{ m}$ and the center frequency $f_c = 60 \text{ GHz}$. In Figure (2.5) the worst case in respect to the attenuation $D$ is shown, this is when the attenuation $D = 1$. The Figure shows that the difference in SNR stabilizes around the $-3 \text{ dB}$ for larger bandwidths and has a minimum of $-4 \text{ dB}$ around bandwidth $B = 0.5 \text{ GHz}$. From simulations it is clear that the introduction of a small delay element only yields a better SNR for small bandwidths.
2.5.4 Two amplifiers and a delay

It is possible to realize a matched filter for the input signal using delay elements. Since the output power at time $t_0$ equals

$$P_1(t_0)_{out} = \left| \int_{f_c - B/2}^{f_c + B/2} F_1(f)S(f)e^{i2\pi ft_0} df \right|^2$$  \hspace{1cm} (2.65)

if the input signal of the filter is

$$S(f) = s_1 + Ds_1 e^{-i2\pi fT_d}.$$  \hspace{1cm} (2.66)

the matched filter can be found with Equation (2.11)

$$F_1(f) = K(s_1^* + Ds_1^* e^{i2\pi fT_d})e^{-i2\pi ft_0}.$$  \hspace{1cm} (2.67)

It is clear from this equation the filter is not realizable at time $t_0 = 0$, since the filter needs to be causal. If therefore the sampling time $t_0$ is chosen sufficiently large $t_0 > T_d$ the filter is physically possible since

$$F_1(f) = K(s_1^* e^{-i2\pi fT_d} + Ds_1^*)e^{-i2\pi ft_0}.$$  \hspace{1cm} (2.68)
where \( t_1 = t_0 - T_d \). From this equation it has become clear a delay \( \delta \) in the filter will delay the sample time from \( t_0 \) to \( t_0 + \delta \). Now a filter with a delay is considered

\[
F_2(f) = (c_1 e^{-i2\pi \delta f} + c_3)e^{-i2\pi f t_0}.
\]  

(2.69)

where \( c_1 \) and \( c_3 \) are complex constants and \( \delta \) is the delay time of the filter. The output power at time \( t_0 \) now equals

\[
P_2(t_0)_{\text{out}} = \left| \int_{f_c-B/2}^{f_c+B/2} F_2(f) S(f) e^{i2\pi f t_0} df \right|^2
\]  

(2.70)

which equals

\[
P_2(t_0)_{\text{out}} = \left| \int_{f_c-B/2}^{f_c+B/2} F_3(f) S(f) e^{i2\pi f t_0} df \right|^2
\]  

(2.71)

where

\[
F_3(f) = (c_1 + c_3 e^{i2\pi \delta f}) e^{-i2\pi f t_0}.
\]  

(2.72)

To find the optimal values of \( c_1 \) and \( c_3 \) in respect to the SNR the filter is defined as in Equation (2.43). The orthogonal space for this case is

\[
e_1 = \frac{1}{\sqrt{B}} \frac{1}{\sqrt{N_0}} e^{iA(\delta)} e^{-i2\pi \delta f} \frac{1}{\sqrt{N_0}} = b_1
\]  

(2.73)

where

\[
A(\delta) = e^{+j2\pi \delta (f_c + B/2)} - e^{+j2\pi \delta (f_c - B/2)}
\]  

(2.74)

By using Equation (2.16) the value of the weights \( a_n = Kh_n^* \) can be derived

\[
h_1 = B b_1 s_1 - \frac{b_1 D s_1}{2\pi T_d} \left( e^{-i2\pi f T_d (f_c + B/2)} - e^{-i2\pi f T_d (f_c - B/2)} \right)
\]  

(2.75)
\[ h_2 = \frac{b_3 D_s_1}{i2\pi(T_d - \delta)} \left( e^{-i2\pi f(T_d - \delta)(f_c + B/2)} - e^{-i2\pi f(T_d - \delta)(f_c - B/2)} \right) \]
\[ - \frac{A(\delta)b_3 D_s_1}{(2\pi)^2T_d\delta B} \left( e^{-i2\pi fT_d(f_c + B/2)} - e^{-i2\pi fT_d(f_c - B/2)} \right) \tag{2.76} \]

The optimized SNR is found with Equation (2.18):

\[ \left( \frac{S}{N} \right)_{out} = |h_1|^2 + |h_2|^2. \tag{2.77} \]

The found optimized SNR for this filter is compared to the matched filter, the filter with only one amplifier and the filter with two amplifiers and a small delay as a function of the delay time \( \delta \) of the filter. Results are shown in Figure (2.6). For simulations the noise was considered to be white noise, the attenuation \( D = \frac{1}{4} \), the path-length \( p = 1 \text{ m} \), the bandwidth \( B = 0.2 \text{ GHz} \) and the center frequency \( f_c = 60 \text{ GHz} \). Figure (2.6) shows that the filter with a delay is equal to the matched filter if the delay time \( \delta \) is equal to the delay time \( T_d \). As was derived in the beginning of this section. The figure also shows that for small delays the filter equals the filter with two amplifiers and a very small delay and for larger delays equals the filter with one amplifier. Figure (2.6) is a cross-section of Figure (2.7). Figure (2.7) shows the \( \Delta \text{SNR} \) as a function of the bandwidth \( B \) and the delay time of the filter \( \delta \). As expected the filter is equal to the matched filter if the delay time \( \delta \) is equal to the delay time \( T_d \) of the signal. Figures (2.8) and (2.9) show the worst-case scenario for the attenuation \( D = 1 \). Figure (2.8) is a cross-section of Figure (2.9) at bandwidth \( B = 0.2 \text{ GHz} \). The figures show that the filter with one delay is matched when \( \delta = T_d \) and that when the filter is not matched the \( \Delta \text{SNR} \) goes to \( -3dB \) for larger bandwidths and has a minimum of \( -4dB \) around bandwidth \( B = 0.5 \text{ GHz} \).

### 2.5.5 Conclusions

The simulations show that for the used channel model a matched filter can be realized using delay elements, as was expected from theory. For small bandwidths a filter with two amplifiers and a small delay yields a better signal to noise ratio than a filter with one amplifier. For larger bandwidths their signal to noise ratios are almost equal.
Figure 2.6: ΔSNR as a function of the delay $\delta$ for $D = \frac{1}{4}$.

Figure 2.7: ΔSNR as a function of the delay $\delta$ and the bandwidth for $D = \frac{1}{4}$. 

F(f) = c_1
F(f) = c_1 + c_2 f
F(f) = c_1 + c_3 e^{-i2 \pi f \delta}$

$0 0.5 1 1.5 2 2.5 3 3.5 \times 10^{-8}$

$-0.2 -0.18 -0.16 -0.14 -0.12 -0.1 -0.08 -0.06 -0.04 -0.02 0$

$d [sec]$

DSNR

$22$
Figure 2.8: $\Delta$SNR as a function of the delay $\delta$ for $D = 1$.

Figure 2.9: $\Delta$SNR as a function of the delay $\delta$ and the bandwidth for $D = 1$. 

23
2.6 Simulating filter-array designs for antenna-array structures

2.6.1 Introduction

Since smart antenna structures are needed for the 60 GHz band, simulations are done on antenna array structures. First the case of two antennas is considered and simulated. Secondly the case of multiple receiver antennas is considered and simulated. To compare different antenna and filter-arrays the difference in signal to noise ratio of the output of the considered optimized filter-array in respect to a matched filter-array is used. The used input signal for the transmitting antenna is a flat-band signal as was described in Section (2.4).

2.6.2 Two receiver antennas

Filters consisting of one amplifier

First the case of two antennas with an amplifier is considered. The antennas are placed in a plane at a distance $L$ in respect to each other and at a distance $R_1$ and $R_2$ in respect to the transmitting antenna (Figure (2.10)),

\[
\begin{align*}
R_1 &= \sqrt{(R + \frac{1}{2}L \cos(\theta))^2 + (\frac{1}{2}L \sin(\theta))^2} \\
R_2 &= \sqrt{(R - \frac{1}{2}L \cos(\theta))^2 + (-\frac{1}{2}L \sin(\theta))^2}
\end{align*}
\]  

(2.78)

where $\theta$ is the angle of the array in respect to the transmitter antenna. The considered filters are of the form

\[F(f) = c_1 e^{-i2\pi f t_0}\]  

(2.79)

The delays of the channel model need to be accounted for. Therefore the power of the signal at time $t_0$ becomes

\[P(t_0) = \int_{-\infty}^{\infty} F(f)H(f)S(f)e^{i2\pi ft_0+\delta} df^2\]  

(2.80)

where $\delta$ is the largest delay of the first signal to reach a given antenna. Figure (2.11) shows $\Delta$SNR as a function of the distance $L$ and the angle $\theta$. The Figure shows that if the distance $L$ between the antennas becomes larger the SNR is more sensitive to the angle $\theta$. The Figure also shows that for the angles where $\theta$ equals 0, $\pi$ or $2\pi$ the lowest values in respect to $L$ correspond to the wavelength
of the bandwidth. In this case the wavelength of the bandwidth $\lambda = 0.03\, m$. For simulations the attenuation $D_1 = D_2 = 1$, the path length $p_1 = 1\, m$, the path length $p_2 = 2\, m$, distance $R = 100\, m$ the bandwidth $B = 10\, GHz$ and the center frequency $f_c = 60\, GHz$. Figure (2.11) is a close up of Figure (2.12). Figure (2.12) shows that when the distance between the receiver antennas $L$ is equal or larger than the path length of one of the delayed signals there are more angles where the SNR has a maximum. In Figure (2.13) the attenuation $D_1 = \frac{1}{4}$ and $D_2 = \frac{1}{4}$. The Figure shows that the local maximums which are due to the reflected signals, are much smaller than they where in Figure (2.12).

**Filters consisting of one amplifier and a delay**

From the previous section it has become clear the spatial distribution of the antennas needs to be accounted for. Therefore an extra delay in the filter is introduced [6]. The considered filters are now of the form

$$F(f)_k = c_{1k} e^{-i2\pi f_{\delta_k}} e^{-i2\pi f_{t_0}}$$

(2.81)

where $\delta_k$ is the extra delay in the filter. The channel model shown in Figure (2.10) is used. Figure (2.14) shows the $\Delta$SNR as a function of the delays in the filters. For simulations the attenuation $D_1 = D_2 = 1$, the path length $p_1 = 1\, m$, the path length $p_2 = 2\, m$, the bandwidth $B = 10\, GHz$, the center frequency $f_c = 60\, GHz$, distance $R = 100\, m$, distance $L = 0.5\, m$ and the angle $\theta = \frac{1}{2}\pi$. The Figure shows that the SNR has a maximum when the delays are equal, as
Figure 2.11: $\Delta$SNR as a function of the angle $\theta$ and the antenna distance $L$.

Figure 2.12: $\Delta$SNR as a function of the angle $\theta$ and the antenna distance $L$. 
Figure 2.13: $\Delta$SNR as a function of the angle $\theta$ and the antenna distance $L$.

was expected from the previous section since the angle $\theta = \frac{1}{2}\pi$. The Figure also shows that there is a maximum in the SNR when the difference in the delays is chosen such that the total delay time of the reflected signal of one antenna is equal to the total delay time of the direct line of sight signal of the other antenna in respect to the transmitting antenna. In Figure (2.15) the angle $\theta = \pi$ and the attenuation $D_1 = D_2 = \frac{1}{4}$. The Figure shows that when the difference between the delays is equal to the difference in the total delay time of the direct line of sight signals, in respect to the transmitting antenna, the SNR is maximal.

2.6.3 Multiple receiver antennas

In the previous section it was found that the spatial distribution can be accounted for by introducing extra delay elements [6]. In this section the extra delays introduced in the previous section are chosen such that the direct line of sight signals all have an equal total delay time in respect to the transmitting antenna. The used channel model is shown in Figure (2.2). For simulations the time delays of the reflected beams are defined as

$$T_{dk} = \frac{k}{c}$$

(2.82)

where $k$ is the $k^{th}$ antenna and $c$ the speed of light. The attenuations of the reflected signals are chosen such that there is a worst case in respect to the SNR, therefore $D_k = 1$. The central frequency $f_c = 60 \, GHz$. The considered filters are a filter consisting of one amplifier and a filter consisting of two amplifiers and a very small delay, each filter is combined with an extra delay to cancel out
Figure 2.14: \( \Delta \text{SNR} \) as a function of the delay \( \delta_1 \) and the delay \( \delta_2 \).

Figure 2.15: \( \Delta \text{SNR} \) as a function of the delay \( \delta_1 \) and the delay \( \delta_2 \).
the effects of spatial distribution. In Figure (2.16) the ∆SNR in respect to the bandwidth is shown for an array where each antenna has a filter consisting of one amplifier. The Figure shows the ∆SNR goes to $-3 \text{dB}$ for larger bandwidths. This can be derived by using Equation (2.47), Equation (2.30) and Equation (2.41),

$$\Delta \text{SNR} = 10 \times \log_{10} \left( \frac{\left( \frac{S}{N} \right)_{\text{out filter}}}{\left( \frac{S}{N} \right)_{\text{out matched filter}}} \right). \quad (2.83)$$

where

$$\left( \frac{S}{N} \right)_{\text{out filter}} = \sum_{k=1}^{K} \frac{|s_k|^2}{2N_0} \left( 1 + \frac{D_k}{\pi T_d B} \cdot \frac{(\sin(2\pi T_d (f_c + B/2)) - \sin(2\pi T_d (f_c - B/2))) + D_k(2 - 2\cos(2\pi T_d B))}{B^2(2\pi T_d B)^2} \right).$$

$$\left( \frac{S}{N} \right)_{\text{out matched filter}} = \sum_{k=1}^{K} \frac{|s_k|^2}{2N_0} \left( 1 + \frac{D_k}{\pi T_d B} \cdot \frac{(\sin(2\pi T_d (f_c + B/2)) - \sin(2\pi T_d (f_c - B/2))) + D_k^2}{B^2(2\pi T_d B)^2} \right). \quad (2.84)$$

If the bandwidth $B$ goes to infinity ∆SNR becomes,

$$\lim_{B \to \infty} \Delta \text{SNR} = 10 \times \log_{10} \left( \frac{\sum_{k=1}^{K} 1}{\sum_{k=1}^{K} 1 + D_k^2} \right). \quad (2.85)$$
Figure 2.17: $\Delta SNR$ as a function of the bandwidth $B$.

Figure 2.18: $\Delta SNR$ as a function of the bandwidth $B$. 

512 antennas: $F_k(f) = c_{1k}$
512 antennas: $F_k(f) = c_{1k} + c_{2k}\alpha f$
Since the attenuations are chosen such that the worst case is considered ($D_k = 1$), $\Delta\text{SNR}$ equals $-3 \, \text{dB}$ for larger bandwidths. In Figure (2.17) the $\Delta\text{SNR}$ in respect to the bandwidth is shown for an array where each antenna has a filter consisting of two amplifiers and a small delay. The Figure shows the $\Delta\text{SNR}$ goes to $-3 \, \text{dB}$ for larger bandwidths. Figure (2.18) shows that for large antenna array structures both filters have the same behavior for larger bandwidths. Since the $\Delta\text{SNR}$ has a value of $-3 \, \text{dB}$ in the worst case scenario of the considered channel model the signal to noise ratio of the output of a matched filter for a given array can be matched or bettered by an array twice the size consisting only out of filters with an amplifier and a delay.

### 2.6.4 Conclusions

The spatial distribution of an antenna array can be accounted for by introducing extra delay elements in the filter design. For the considered channel model the signal to noise ratio of a matched filter for a given array can be matched or bettered by an array twice the size consisting only out of filters with an amplifier and a delay.

### 2.7 Conclusions

Analytical models have been derived to characterize a single filter and a filter-array structure as such that the signal to noise ratio is maximal. For the case of one transmitter and one receiver antenna a matched filter can be realized using two amplifiers and a delay element. For small bandwidths a filter with two amplifiers and a small delay yields a better signal to noise ratio than a filter with one amplifier. For larger bandwidths their signal to noise ratios are almost equal. For the case of a receiver antenna-array the spatial distribution of an antenna-array structure can be accounted for by introducing extra delay elements in the filter design. Also the signal to noise ratio of a matched filter for a given array can be matched or bettered by an array twice the size consisting only out of filters with an amplifier and a delay.
Chapter 3

Radiation patterns

3.1 Introduction

To get an insight into the effects of beam-steering the radiation pattern of an isotropic antenna array system is derived in Section (3.2). Since isotropic antennas are not a physicality but an ideality, the isotropic antenna elements are replaced by realistic antenna elements. There are several options for 60 GHz antennas. Due to production-cost and chip-integration considerations, the antenna focused on in this report is the rectangular patch antenna [8, 9, 10, 11, 12, 13]. To derive the number of patch antennas needed for transmission the total required array gain needs to be calculated. From literature [1] it is known the legal transmit power limit is $20 \text{dBm}$. It is also known that the input noise power at the receiver is $10 \log(kTBF)$ where $k$ is Boltzmann’s constant ($k = 1.38 \cdot 10^{-23}$), $T$ is the equivalent noise temperature of the receiver ($T = 290^\circ K$), $B$ is the noise bandwidth and $F$ is the receiver noise figure. With a receiver noise figure of $10 \text{dB}$ and a noise bandwidth of $1 \text{GHz}$ the receiver noise power amounts to $-104 \text{dB}$. For sufficient performance in terms of bit error ratio ($< 10^{-6}$) an SNR of about $10 \text{dB}$ is required. According to Friis formula a $10 \text{dB}$ antenna gain is needed at the transmitter and the receiver. A typical rectangular patch antenna has a gain of $3 \text{dB}$. From the previous Chapter it is known that due to the non ideality of the analog filter the SNR can drop $3 \text{dB}$. The gain of the antenna array system therefore needs to be at least $10 \text{dB}$, therefore 16 antennas are needed. The 16 antenna patch array combined with the analog filters give an increase in the SNR of $3 - 3 + 12 = 12 \text{dB}$. To obtain a more uniform performance of the array in all directions, symmetrical array structures are advised. The radiation pattern of a rectangular patch antenna is introduced in Section (3.3). In Section (3.4) the radiation pattern of a patch antenna array is derived. Simulations are done in Section (3.5), Section (3.6) and Section (3.7).
3.2 Isotropic antenna arrays

3.2.1 Introduction

An isotropic antenna is an antenna which radiates equally in all directions [17]. Although such an antenna is not a physicality but an ideality it is used in this chapter to calculate the radiation pattern of array structures, the "group pattern" or "array factor" [17]. It should be noted that in the previous Chapter the antennas were considered to be isotropic. In this Section the considered structure is a flat array with \( N \times M \) elements.

3.2.2 Radiation pattern of an Isotropic antenna array

First an array of only \( N \) elements is considered. The array is assumed linear in that the elements are spaced equally along a straight line and lie along the \( x\)-axis (Figure 3.1). Also it is assumed that the array is uniform so that each element is fed with a current of the same magnitude, but with a progressive phase shift \( \alpha_x \). The far field can now be written as

\[
|E| = |E_0(1 + e^{j\psi_1} + e^{j2\psi_1} + e^{j3\psi_1} + \ldots + e^{jN\psi_1})| \quad (3.1)
\]

where
\[ \psi_x = \beta d_x \sin \theta + \alpha_x \] (3.2)

with \( \beta = 2\pi/\lambda \), \( d_x \) is the spacing between the elements, \( \alpha_x \) is the inter-element phase shift and \( \lambda \) is the wavelength of the current. Equation (3.1) can be rewritten as

\[
|E| = \left| E_0 \frac{1 - e^{jN\psi_x}}{1 - e^{j\psi_x}} \right| (3.3)
\]

which can be written as

\[
|E| = \left| E_0 e^{jN\psi_x/2} \frac{(e^{-jN\psi_x/2} - e^{jN\psi_x/2})}{e^{j\psi_x/2}(e^{-j\psi_x/2} - e^{j\psi_x})} \right| (3.4)
\]

or

\[
|E| = \left| E_0 e^{j(N-1)\psi_x/2} \frac{\sin(N\psi_x/2)}{\sin(\psi_x/2)} \right|. (3.5)
\]

The phase factor \( e^{j(N-1)\psi_x/2} \) corresponds to a distance \((N-1)d_x/2\), which is the center of the antenna array. If therefore \( E_0^* \) is the field coming from the center of the array the array factor can be written as

\[
\left| \frac{E}{E_0^*} \right| = \left| \frac{\sin(N\psi_x/2)}{\sin(\psi_x/2)} \right| (3.6)
\]

or the normalized array factor for this array

\[
\left| \frac{E}{N E_0^*} \right| = \left| \frac{\sin(N\psi_x/2)}{N \sin(\psi_x/2)} \right|. (3.7)
\]

The array can be expanded to be a two dimensional array of \( N \times M \) elements. Where \( N \) represents the number of elements in the \( x \)-direction and \( M \) the number of elements in the \( y \)-direction. The total array factor can be seen as \( M \) antennas on the \( y \)-axis with a radiation pattern equal to the array factor of the \( N \) elements along the \( x \)-axis [17]. Therefore the total array factor can be written in spherical coordinates (Figure 3.2) as

\[
\left| \frac{E}{N M E_0^*} \right| = \left| \frac{\sin(N\psi_x/2)}{N \sin(\psi_x/2)} \right| \left| \frac{\sin(M\psi_y/2)}{M \sin(\psi_y/2)} \right| (3.8)
\]
where

\[ \psi_x = \beta d_x \sin \theta \cos \phi + \alpha_x \]
\[ \psi_y = \beta d_y \sin \theta \sin \phi + \alpha_y \]  

(3.9)

where \( \alpha_x \) is the inter-element phase shift in the \( x \)-direction, \( \alpha_y \) is the inter-element phase shift in the \( y \)-direction and \( \mathbf{E}_0^c \) is the field coming from the center of the array.

### 3.2.3 Conclusions

A model to calculate the "array factor" of a flat array with \( N \times M \) elements has been derived and will be used in Section (3.5) for simulations.

### 3.3 Patch antenna

#### 3.3.1 Introduction

The antenna focused on in this section is the rectangular patch antenna. To model the normalized radiation pattern of a patch antenna the cavity model is used. It should be noted that the range of accuracy of the cavity model is limited and it is advised to use other models for more complex calculations. However, the cavity model is useful to calculate the normalized radiation pattern and gives good insight in the physics of a patch antenna.

As seen in [18] a patch antenna is constructed out of a dielectric with a conductive ground-plane underneath. On top of the dielectric is a conductive strip. The conductive strip on top of the dielectric is called the patch (Figure (3.3)). The width \( a \) and length \( b \) of the antenna typically lie around \( \lambda/2 \), where \( \lambda \) is
the wavelength of the operating frequency. The height \( h \) of the antenna is a small fraction of the wavelength. The patch conductor is normally copper and can take any shape, but generally simple geometries are used. This simplifies the analysis and performance prediction. Usually the patches are photo etched on the dielectric substrate.

There are different techniques to feed the patch antenna. The method used in this study is a coaxial feed line. A coaxial probe feeds the patch on the backside of the antenna. The inner conductor of the coaxial line is connected to the conducting strip and the outer conductor is connected to the conductive ground-plane (Figure (3.3)). Other feed technologies are for example a coplanar feed line, where the feed line is on the same substrate as the patch and is directly connected to the patch. Another alternative is an electromagnetically coupled feed line, where the feed line is not directly connected to the patch, but is electromagnetically coupled to the patch [19].

Between the patch and the ground-plane the electric fields are perpendicular to the conductive planes. Near the edges of the patch the field bends outwards, these are called the "fringe fields".

### 3.3.2 Cavity model

To determine the resonance frequency of the patch the cavity model is used. In this model the antenna is considered as a resonating cavity. This implies that the patch antenna is viewed as a box, with perfect electric conducting ground and top plane equal in size to the patch [20]. The side walls are assumed to be magnetic conductors. The "fringe fields" are taken into account by considering a slightly larger box shown in Figure (3.4). Because of the boundary conditions only electric fields exist which are perpendicular to the conducting planes. The magnetic field has a vanishing tangential component at the four side walls. The fields of the lowest radiating mode (assuming \( a \geq b \)) are given by:
Figure 3.4: Effective width of a patch antenna.

\[
E_z(x) = -E_0 \cos\left(\frac{\pi x}{a}\right) \text{ for } 0 \leq x \leq a
\]
\[
H_y(x) = -H_0 \sin\left(\frac{\pi x}{a}\right) \text{ for } 0 \leq x \leq a
\]

(3.10)

This means that the dimensions of the patch dictate the modes that can exist in this perfect cavity. The lowest possible frequency is often the frequency at which the patch is used.

Consider a rectangular patch of width \(a\) and length \(b\) over an infinite ground-plane with a substrate of thickness \(h\) and a dielectric constant \(\varepsilon_r\), as shown in figure (3.3). As long as the substrate is electrically thin, the electric field will be \(z\)-directed and the interior modes will be \(TM_{mn}\) to \(z\) so that according to the cavity model [13]:

\[
E_z(x, y) = \sum_m \sum_n A_{mn} \psi_{mn}(x, y),
\]

(3.11)

where \(A_{mn}\) are the mode amplitude coefficients and \(\psi_{mn}\) are the \(z\)-directed orthonormalized electric field mode vectors. For the elementary case of a non-radiating cavity,

\[
\psi_{mn}(x, y) = \cos(k_m x) \cos(k_n y),
\]

(3.12)

with

\[
k_m = \frac{m\pi}{a} \text{ and } k_n = \frac{n\pi}{b}.
\]

(3.13)

The resonance condition is given by:

\[
k^2 = k_m^2 + k_n^2 = k_{mn}^2,
\]

(3.14)

where the propagation constant \(k = \frac{\sqrt{\varepsilon_r^2 \pi^2}}{\lambda_0}\). Here \(\lambda_0\) represents the wavelength of the fields in free space. The amplitude coefficients are given by:
\begin{equation}
A_{mn} = \frac{j\chi_{mn}^2}{abh} \frac{\omega \mu}{k^2 - k_{mn}^2} \int_V J_z \psi_{mn}(x,y) dV,
\end{equation}

with \( \mu \) the permeability of the dielectric, \( J_z \) the current source, \( V \) the volume of the patch and \( \omega = 2\pi f \) with \( f \) the frequency. Here \( \chi_{mn} \) is given by:

\begin{equation}
\chi_{mn} = \begin{cases} 
1, & m = 0 \text{ and } n = 0 \\
\sqrt{2}, & m = 0 \text{ or } n = 0 \\
2, & m \neq 0 \text{ and } n \neq 0 
\end{cases}.
\end{equation}

If it is assumed the patch antenna has a probe feed at point \( x_p \) and \( y_p \) with negligible diameter, the current source \( J_z \) can be modelled as \( J_z = I_0 \delta(x - x_p)\delta(y - y_p) \). Here \( I_0 \) can be chosen constant because of the limited height of the substrate. The amplitude coefficients are then given by:

\begin{equation}
A_{mn} = jI_0 \frac{\eta \chi_{mn}^2}{abh} \frac{k}{k^2 - k_{mn}^2} \psi_{mn}(x_p, y_p),
\end{equation}

where \( \eta = \sqrt{\frac{\mu}{\varepsilon}} \) represents the intrinsic impedance.

In practise the dielectric material is not perfectly isolating, the patch and ground-plane are not perfectly conducting and the cavity has no side walls which are perfect magnetic conductors.

The absence of the side walls means that the antenna radiates from these sides. Parameters that influence the radiation properties of the antenna, next to the dimensions of the cavity, are the frequency, the conductivity of the patch and ground-plane, the dielectric constant of the dielectric and the loss tangent of the dielectric.

### 3.3.3 Radiation pattern

In the previous section the electric field inside the patch has been presented. From this calculated electric field it is possible to derive the far field pattern. This is done by using the equivalent magnetic current density \( \vec{M} = \vec{E} \times \vec{n} \) [21] on the side walls of the antenna according to Love’s equivalence principle [22]. This is illustrated in Figure (3.5).

From the equivalent magnetic current density on the side walls the electric potential vector for a side wall can be derived [23]

\begin{equation}
\vec{F} = \frac{\varepsilon_0}{4\pi} \int_S \vec{M}_s e^{-jk_{mn}|\vec{R}|} dS',
\end{equation}
Figure 3.5: Electric field distribution (a) and equivalent magnetic current density (b).

where \( \varepsilon_0 \) represents the permittivity of free space and \( k_0 = \frac{2\pi}{\lambda_0} \) the propagation constant. Here \( \lambda_0 \) represents the wavelength of the fields in free space. the surface \( S \) represents the side wall and \( |\vec{R}| \) the distance between the source and observation point.

To determine the radiation pattern of a rectangular patch antenna each side is assumed to be an independent radiating aperture. The expression for the radiated electric field in the far field region for the side \( x = 0 \) given in spherical coordinates (Figure 3.2)) is:

\[
\vec{E}_{rad} = -j \frac{k_0}{\varepsilon_0} F_y (\cos(\theta) \sin(\phi) \vec{a}_\phi + \cos(\phi) \vec{a}_\theta),
\]

(3.19)

where

\[
\vec{F} = j \frac{\varepsilon_0 bh \vec{a}_y}{\pi r} e^{-jk_0 r} A_{mn} \frac{Y[(-1)^n e^{-j2Y} - 1]}{4Y^2 - n^2\pi^2} \text{sinc}(Z) e^{-jZ},
\]

(3.20)

and

\[
Y = \frac{b}{2} k_0 \sin(\theta) \sin(\phi),
\]

\[
Z = \frac{L}{2} k_0 \cos(\theta).
\]

(3.21)

Every opposite side has an equal radiation pattern with a phase difference due to the mode and the distance between the sides. the total radiated field can be calculated by considering the antenna as two arrays of two aperture antennas. The array factor for two identical antennas with phases \( \alpha_1, \alpha_2 \) and a vectorial spacing \( d \) is given by [17]
AF = 2\cos\left(\frac{k_0}{2} \vec{d} \cdot \vec{r} + \frac{\alpha_1 - \alpha_2}{2}\right)e^{j(\alpha_1 - \alpha_2)}.

Thus the total radiated field is given by

$$\vec{E}_{rad} = \vec{E}_{rad}^a AF_a + \vec{E}_{rad}^b AF_b,$$

with

$$\vec{E}_{rad}^a = -jk_0 \frac{b}{h} e^{-jk_0 r} A_{mn} Y \frac{1}{4X^2 - m^2 \pi^2} \sin(Z) e^{-jZ} (\cos(\theta) \sin(\phi) \vec{a}_\phi + \cos(\phi) \vec{a}_\theta),$$

$$\vec{E}_{rad}^b = -jk_0 \frac{a}{h} e^{-jk_0 r} A_{mn} X \frac{1}{4X^2 - m^2 \pi^2} \sin(Z) e^{-jZ} (\cos(\theta) \cos(\phi) \vec{a}_\phi - \sin(\phi) \vec{a}_\theta),$$

and

$$AF_a = 2\cos(X - \alpha_m)e^{j\alpha_m},$$

$$AF_b = 2\cos(X - \alpha_n)e^{j\alpha_n},$$

where

$$X = \frac{a}{h} k_0 \sin(\theta) \cos(\phi),$$

$$Y = \frac{b}{h} k_0 \sin(\theta) \sin(\phi),$$

$$Z = \frac{h}{k_0} \cos(\theta),$$

and

$$\alpha_i = \begin{cases} \frac{\pi}{2}, & \text{for } i \text{ even} \\ \frac{\pi}{2}, & \text{for } i \text{ odd} \end{cases}.$$

The radiated field for multiple modes is found by summing the radiated fields of the independent modes.

The total radiated power for the mode with indices $mn$ can be found by integrating the square of the radiated field over half a hemisphere, since the antenna does not radiate below the infinite ground plane,

$$P_{rad} = \int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=0}^{2\pi} \frac{1}{2k_0} |\vec{E}_{rad}|^2 r^2 \sin(\theta) d\phi d\theta,$$
with \( \eta_0 = \sqrt{\mu_0 / \varepsilon_0} \) the wave impedance for vacuum.

### 3.3.4 Conclusions

A model to calculate the radiated field of a rectangular patch antenna has been derived. The model is used to calculate the normalized electric field in Section (3.6) and Section (3.7). It should be noted that the model is only accurate for \(-80^\circ \leq \theta \leq 80^\circ\).

### 3.4 Patch antenna arrays

#### 3.4.1 Introduction

To model the normalized gain pattern of the far field of an antenna array, first the normalized electric field must be derived. To find the normalized electric field of a patch antenna array, the "group pattern" or "array factor" and the normalized radiation pattern of a single antenna element at the origin, the "unit pattern" or "element factor", can be used. Another method is to progressively place the antenna element at a different location and to sum up the resulting fields. If the antenna element is orientated in the same direction at every position both methods yield the same result. If the antenna element is not orientated in the same direction only the second model can be used. It should be noted that the methods can only be used to predict the normalized field pattern, since coupling between the antenna elements is neglected. This coupling can however cause a significant loss in antenna gain if the array is not directed in the z-direction.

#### 3.4.2 Radiation pattern

**Array and element factor**

The first method to model the normalized radiation pattern of an antenna array is by multiplying the "group pattern" with the "unit pattern" the result is the electric field of the array [17]. This method is known as *pattern multiplication*. In the case of a patch antenna the equations becomes

\[
E_{\text{total}}^{\text{norm}}(\theta, \phi) = E_{\text{patch}}^{\text{norm}}(\theta, \phi) \times E_{\text{array factor}}^{\text{norm}}(\theta, \phi). \tag{3.29}
\]

The normalized gain pattern of the far field is
\[ P_{rad}^{\text{norm}}(\theta, \phi) = |E_{\text{total}}^{\text{norm}}(\theta, \phi)|^2. \] (3.30)

Field summation

The second method is to summate the field of all the different antennas in the considered point. The total electric field will be

\[ E_{\text{total}}^{\text{norm}}(\theta, \phi) = \frac{1}{K} \sum_{k=1}^{K} E_{\text{patch}_k}^{\text{norm}}(\theta, \phi) \] (3.31)

where \( K \) is the total number of antennas and \( E_{\text{patch}_k}(\theta, \phi) \) is the field of the \( k^{th} \) antenna at the considered point. The normalized gain pattern of the far field is

\[ P_{rad}^{\text{norm}}(\theta, \phi) = |E_{\text{total}}^{\text{norm}}(\theta, \phi)|^2. \] (3.32)

3.4.3 Conclusions

Two models to calculate the normalized gain pattern of a patch antenna array have been introduced. The models are used for simulations in Section (3.7).
3.5 Simulating isotropic antenna arrays

3.5.1 Introduction

The isotropic antenna array can be compared to the filter- and antenna-array systems that were described in the previous Chapter. In which every antenna has its own filter and the filter consists out of an amplifier and a delay. For the elementary case that all amplifiers are set to one and the delays correspond to the spatial distribution of the array, the "array factor" gives the field distribution of the simple filter- and antenna-array system of the previous Chapter.

3.5.2 Radiation pattern

For simulations the normalized gain pattern was used, where

\[ P_{\text{rad}}^{\text{norm}}(\theta, \phi) = |E_{\text{total}}^{\text{norm}}(\theta, \phi)|^2, \]  

(3.33)

which is equal to

\[ P_{\text{rad}}^{\text{norm}}(\theta, \phi) = \frac{\sin(N\psi_x/2)}{N \sin(\psi_x/2)} \cdot \frac{\sin(M\psi_y/2)}{M \sin(\psi_y/2)} \]  

(3.34)

where

\[ \psi_x = \beta d_x \sin \theta \cos \phi + \alpha_x, \]

\[ \psi_y = \beta d_y \sin \theta \sin \phi + \alpha_y, \]  

(3.35)

where \( \beta = 2\pi/\lambda \), \( N \) represents the number of elements in the \( x \)-direction, \( M \) is the number of elements in the \( y \)-direction, \( d_x \) is the spacing between the elements in the \( x \)-direction, \( d_y \) is the spacing between the elements in the \( y \)-direction, \( \alpha_x \) is the inter-element phase shift in the \( x \)-direction, \( \alpha_y \) is the inter-element phase shift in the \( y \)-direction, and \( \lambda \) is the considered wavelength.

From Equation (3.34) it is shown that the gain pattern of the antenna array is a function of the number of elements, the spacing between the elements, the inter-element phase shift and the considered wavelength. To show the effects of this parameters on the gain pattern different simulations have been done. For simulations a standard square array of \( 4 \times 4 \) elements at a distance \( d_x = d_y = 1/2\lambda \) operating at a frequency of 60 GHz is used.
Figure 3.6: Normalized gain pattern of a $4 \times 4$ isotropic antenna array with progressive inter-element distance $d$ ($d = d_x = d_y$): (a) $d = 0.5\lambda$, (b) $d = 1.0\lambda$, (c) $d = 1.5\lambda$, (d) $d = 2.0\lambda$.

**Element distance**

First the element distance between the isotropic antennas is progressively increased. The orientation of the array is set in the $z$-direction, therefore the inter-element phase shift in the $x$- and $y$-direction is set to zero. The inter-element distance in the $x$- and $y$-direction is equal, the general inter-element distance $d$ is defined as $d = d_x = d_y$. The operating frequency is $60\,GHz$.

Figure (3.6) shows that when the distance between the antenna elements is being increased more "beams", next to the desired beam in the $z$-direction, are being introduced to the gain pattern. The introduced "beams" next to the desired beam are called "side lobes". To prevent the introduction of side lobes in the antenna gain pattern, the inter-element distance should not be too large. On the other hand if the distance between the antenna elements becomes too small "beam forming" no longer occurs. Therefore it is advised to keep the inter-element distance in the $x$-direction $d_x$ between $1/2\lambda < d_x < \lambda$ and in the $y$-direction $d_y$ between $1/2\lambda < d_y < \lambda$. where $\lambda$ represents the wavelength of the signal in free-space.
Number of elements

Secondly the number of elements is changed. The number of elements is only changed in the \(x\)-direction. The orientation of the array is set in the \(z\)-direction, therefore the inter-element phase shift in the \(x\)- and \(y\)-direction is set to zero. The inter-element distance in the \(x\)- and \(y\)-direction is equal and is set to \(d_x = d_y = 1/2\lambda\). The array is operating at 60\(\text{GHz}\). Figure (3.7) shows that with an increase in the number of elements in the \(x\)-direction, the width of the ”beam” in the \(\phi = 0^\circ\) plane decreases. The ”half-power beamwidth” \(\theta_{\text{half}}\) is defined as the difference in the angles at which the normalized gain has decreased from 0\(\text{dB}\) to \(-3\text{dB}\) for \(\theta_b < (\theta_b + 90^\circ)\) in the \(\phi = \phi_b\) plane, where \(\theta_b\) and \(\phi_b\) are defined as the angles where \(P_{\text{rad}}^{\text{norm}}(\theta_b,\phi_b) = 1\). In the considered case \(\phi_b = 0^\circ\) and \(\theta_b = 0^\circ\). For the considered array the \(-3\text{dB}\) angles can be found with the use of

\[
\left| \frac{\sin(N\pi/2\sin(\theta))}{N\sin(\pi/2\sin(\theta))} \right|^2 = \frac{1}{2} \tag{3.36}
\]

where \(N\) is the number of antenna elements. In Figure (3.7) the ”half-power beamwidth” \(\theta_{\text{half}}\) equals for each case: (a) \(\theta_{\text{half}} = 60^\circ\), (b) \(\theta_{\text{half}} = 26^\circ\), (c) \(\theta_{\text{half}} = 14^\circ\), (d) \(\theta_{\text{half}} = 10^\circ\).

Inter-element phase shift

Thirdly the inter-element phase shift is changed. The considered array is a \(4 \times 4\) array, the element distance is \(\lambda/2\), the operating frequency of the array is 60\(\text{GHz}\). In Figure (3.8) the normalized gain pattern in the \(\phi = 0^\circ\) plane for different progressive inter-element phase shifts in the \(x\)-direction is shown. The inter-element phase shift in the \(y\)-direction is set to zero. The figure shows that the ”beam” widens when the phase shift approaches \(\alpha_x = -1.0\pi\). Figure (3.9) shows the normalized gain pattern in the \(\phi = 45^\circ\) plane for an equal progressive inter-element phase shifts in the \(x\)- and \(y\)-direction. Figure (3.9) shows that the gain is reduced when \(\alpha_x = \alpha_y = -1.0\pi\). From simulations it has become clear that a normalized gain of value one can be attained in every direction. It also became clear that the ”half-power beamwidth” and the value of the gain are influenced by the inter-element phase shift.

3.5.3 Conclusions

From simulations it has become clear that the number of ”side lobes” is a function of the distance between the antenna elements. The ”half-power beamwidth” can be reduced by introducing more antenna elements. The angle of the beam can be controlled with the inter-element phase shift.
Figure 3.7: Normalized gain pattern ($\phi = 0^\circ$) of an $N \times 4$ isotropic antenna array with number of elements $N$ in the $x$-direction: (a) $N = 2$, (b) $N = 4$, (c) $N = 8$, (d) $N = 16$. 
Figure 3.8: Normalized gain pattern ($\phi = 0^\circ$) of an $4 \times 4$ isotropic antenna array with progressive inter-element phase shift: (a) $\alpha_x = 0.0\pi$, (b) $\alpha_x = -0.5\pi$, (c) $\alpha_x = -1.0\pi$, (d) $\alpha_x = -1.5\pi$. 
Figure 3.9: Normalized gain pattern ($\phi = 45^\circ$) of an $4 \times 4$ isotropic antenna array with progressive inter-element phase shift $\alpha = \alpha_x = \alpha_y$: (a) $\alpha = 0.0\pi$, (b) $\alpha = -0.707\pi$, (c) $\alpha = -1.0\pi$, (d) $\alpha = -1.293\pi$. 
3.6 Simulating a patch antenna

3.6.1 Introduction

Until now only isotropic antenna elements where considered in simulations. Since isotropic antenna elements are an ideality and not a physicality a more realistic antenna element will be considered. Due to production-cost and chip-integration considerations, the antenna focused on in this report is the rectangular patch antenna [8, 9, 10, 11, 12, 13]. To calculate the normalized radiated field of a patch antenna, the cavity model is used. It should be noted that the range of accuracy of the cavity model is limited and it is advised to use other models for more complex calculations. However, the cavity model is useful to calculate the normalized radiation pattern of a patch antenna.

3.6.2 Radiation pattern

If the antenna element is isotropic, the beam of the antenna array can be directed in every direction. To be able to beam in as much directions as possible it is necessary to have a normalized antenna gain pattern, which is as omnidirectional as possible. Therefore simulations where done in which the size of the patch and the position of the feed-point where changed.

For simulations a patch was used with dimensions: width $a = \lambda/2$, length $b = \lambda/2$, height $h = 0.0268\lambda$, where $\lambda$ is the wavelength in the dielectric. The dielectric constant of the dielectric $\varepsilon_r = 4.28$ and the loss tangent $\tan\delta = 1.6 \cdot 10^{-2}$. The operating frequency corresponds to 60 GHz. In Figure (3.10) for case (a) the probe feed position is $x_p = a/4, y_p = b/2$, for case (b) the probe feed position is $x_p = a/2, y_p = b/4$ and for case (c) the probe feed position is $x_p = a/4, y_p = b/4$. From literature [18] it is known a longer patch can transmit more power, therefore simulations have been done on longer patches. In Figure (3.10) for case (d) the length of the patch $b = 5\cdot\lambda/2$, the probe feed position is $x_p = a/4, y_p = b/2$. The dimensions of the patch where chosen such that the fundamental mode $TM_{10}$ or $TM_{01}$ is dominant. This is done because higher order modes introduce unwanted side lobes. The normalized gain pattern as shown in Figure(3.10a) is dominated by the fundamental mode $TM_{10}$, in Figure(3.10b) by the fundamental mode $TM_{01}$, in Figure(3.10c) by the fundamental modes $TM_{10}$ and $TM_{01}$ and in Figure (3.10d) by the fundamental mode $TM_{10}$. In Figure (3.10) can be seen that a longer patch results in a “flatter” normalized radiation pattern in respect to the square patch antenna. Therefore a shorter antenna is preferred. On the other hand the length of the antenna should not become too small, since then the ability of the patch to transmit power can become too small. Therefore a square patch is preferred over a rectangular patch. From simulations it became clear that the best option for an antenna element in the antenna array is a square patch antenna with a probe feed position $x_p < a/2, y_p = b/2$ (where $TM_{10}$ is the dominant mode) or $x_p = a/2, y_p < b/2$ (where $TM_{01}$ is the dominant mode) or along the diagonal.
Figure 3.10: Normalized gain patterns of a patch antenna.
3.6.3 Conclusions

From simulations it has become clear that a square patch antenna dominated by the fundamental mode \(TM_{10}\), \(TM_{01}\) or both is the best option for an antenna element in the antenna array. The position of the probe feed dictates which mode is dominant.

3.7 Simulating patch antenna arrays

3.7.1 Introduction

The radiation pattern of the antenna element changes the radiation pattern of the entire antenna array. To get an insight into this changes the normalized gain pattern of a patch antenna array is presented. It should be noted that in the simulations the coupling between the antenna elements has been neglected. This coupling can however cause a significant loss in antenna gain if the array is not directed in the \(z\)-direction. Therefore the effects of coupling on the ability of antenna arrays to "steer" the "beam" deserves further investigation. In this report however the effects have been neglected.

3.7.2 Radiation pattern

If the antenna elements are all orientated in the same direction there are effectively two options for the antenna elements. The first option is the square patch antenna with a probe feed position on the diagonal. The second option is a square patch antenna with a probe feed position \(x_{p1} < a/2, y_{p1} = b/2\) (where \(TM_{10}\) is the dominant mode) or \(x_{p2} = a/2, y_{p2} < b/2\) (where \(TM_{01}\) is the dominant mode), which both yield the same results except for their orientation when \(x_{p1} = y_{p2}\) and \(y_{p1} = x_{p2}\). Therefore only the case with a probe feed position \(x_{p1} < a/2, y_{p1} = b/2\) (where \(TM_{10}\) is the dominant mode) is considered. The angles until which the array is able to "beam" are defined as the angles at which the normalized gain of the main lobe has decreased from 0 \(dB\) until \(-3 dB\).

First the square patch antenna with a probe feed position on the diagonal is simulated. It will be used as an antenna element in a \(4 \times 4\) array. In the simulations the angle at which the normalized gain equals \(-3 dB\) was calculated. The results can be found in Figure (3.11) and Figure (3.12). Figure(3.11) shows that the angles at which the normalized antenna array gain of the main lobe equals \(-3 dB\) are the same in the \(\phi = 0^\circ\) plane and the \(\phi = 90^\circ\) plane. These angles are \(-40^\circ\) and \(40^\circ\). Figure (3.12) shows that for the \(\phi = 45^\circ\) plane and
Figure 3.11: Normalized gain patterns of a patch antenna array where the progressive inter-element phase shift equals: (a) \((\phi = 0^\circ) \alpha_x = -0.708\pi, \alpha_y = 0\); (b) \((\phi = 0^\circ) \alpha_x = -1.297\pi, \alpha_y = 0\); (c) \((\phi = 90^\circ) \alpha_x = 0, \alpha_y = -0.708\pi\); (d) \((\phi = 90^\circ) \alpha_x = 0, \alpha_y = -1.297\pi\).

The \(\phi = 135^\circ\) plane the angles differ. In the \(\phi = 45^\circ\) plane the angles are \(-27^\circ\) and \(27^\circ\). In the \(\phi = 135^\circ\) plane the angles are \(-48^\circ\) and \(48^\circ\).

Secondly the square patch antenna with a probe feed position \(x_{p1} < a/2, y_{p1} = b/2\) (where \(TM_{10}\) is the dominant mode) is simulated. It will be used as an antenna element in a \(4 \times 4\) array. In the simulations the angle at which the normalized gain equals \(-3\,dB\) was calculated. The results can be found in Figure (3.13) and Figure (3.14). Figure (3.13) shows that the angles at which the normalized antenna array gain of the main lobe equals \(-3\,dB\) are different in the \(\phi = 0^\circ\) plane and the \(\phi = 90^\circ\) plane. The figure shows that in the \(\phi = 0^\circ\) plane the normalized antenna array gain of the main lobe never equals \(-3\,dB\). The antenna array can beam in this plane between the angles \(-80^\circ\) and \(80^\circ\). This is not \(-90^\circ\) and \(90^\circ\) because the cavity model, which is used, is only accurate for \(-80^\circ \leq \theta \leq 80^\circ\). In the \(\phi = 90^\circ\) plane the angles are \(-25^\circ\) and \(25^\circ\). Figure (3.14) shows that the angles of the \(\phi = 45^\circ\) plane and the \(\phi = 135^\circ\) plane are the same. These angles are \(-38^\circ\) and \(38^\circ\).

From the two simulations it has become clear that an antenna array consisting out uniformly orientated square patch antennas with a probe feed on the diagonal has a better performance in most angles compared to an array consisting
Figure 3.12: Normalized gain patterns of a patch antenna array where the progressive inter-element phase shift equals: (a) ($\phi = 45^\circ$) $\alpha_x = \alpha_y = -0.399\pi$; (b) ($\phi = 45^\circ$) $\alpha_x = \alpha_y = -1.602\pi$; (c) ($\phi = 135^\circ$) $\alpha_x = -1.425$, $\alpha_y = -0.575\pi$; (d) ($\phi = 135^\circ$) $\alpha_x = -0.575\pi$, $\alpha_y = -1.425\pi$. 
Figure 3.13: Normalized gain patterns of a patch antenna array where the progressive inter-element phase shift equals: (a) $(\phi = 0^\circ) \alpha_x = -1\pi, \alpha_y = 0$; (b) $(\phi = 0^\circ) \alpha_x = -1\pi, \alpha_y = 0$; (c) $(\phi = 90^\circ) \alpha_x = 0, \alpha_y = -0.531\pi$; (d) $(\phi = 90^\circ) \alpha_x = 0, \alpha_y = -1.469\pi$. 
Figure 3.14: Normalized gain patterns of a patch antenna array where the progressive inter-element phase shift equals: (a) ($\phi = 45^\circ$) $\alpha_x = \alpha_y = -0.491\pi$; (b) ($\phi = 45^\circ$) $\alpha_x = \alpha_y = -1.513\pi$; (c) ($\phi = 135^\circ$) $\alpha_x = -1.515\pi$, $\alpha_y = -0.485\pi$; (d) ($\phi = 135^\circ$) $\alpha_x = -0.490$, $\alpha_y = -1.510\pi$. 
out of uniformly orientated square patch antennas with a probe feed along the central axis \( (x_p = a/2 \text{ or } y_p = b/2) \) of the patch. Although the array consisting out of patch antennas with a probe feed along the central axis of the patch antenna has a better performance along that axis. An antenna array consisting out of differently orientated patches could be considered to enhance the \(-3dB\) angles of the entire array structure. An other alternative is a patch array where the feed point of the patch can be chosen.

### 3.7.3 Conclusions

An antenna array consisting out uniformly orientated square patch antennas with a probe feed on the diagonal has a better performance in most angles compared to an array consisting out of uniformly orientated square patch antenna with a probe feed along the central axis \( (x_p = a/2 \text{ or } y_p = b/2) \) of the patch. Although the array consisting out of patch antennas with a probe feed along the central axis of the patch antenna has a better performance along that axis.

### 3.8 Conclusions

Models to calculate the radiation patterns of isotropic arrays, rectangular patch antennas and patch antenna arrays have been derived. From simulations it became clear that the antenna elements should not be spaced too far apart to prevent the introduction of unwanted side-lobes. It also became clear that the main effect of the patch antenna element is a limitation to the angles at which the antenna array can "beam". The preferred antenna element is a square patch antenna. The position of the feed point changes the ability of the array to beam in certain angles. An antenna array consisting out of differently orientated patches could be considered to enhance the ability to "beam" in more angles. An other alternative is a patch array where the feed point of the patch can be chosen.
Chapter 4

Conclusions and recommendations

According to the simulations the signal to noise ratio of an antenna array which is matched for the input signal can be matched or bettered by an array twice the size consisting only out of filters with an amplifier and a delay. Taking into account that a simpler filter design reduces significantly the complexity of the algorithm needed to optimize the signal to noise ratio, since the filters can be optimized separately, it is preferred over a more complex solution. It should be noted that in the channel model every receiving antenna has a direct line of sight signal and a reflected signal from a surface, where the reflected signal is considered to be smaller or equal to the direct line of sight signal.

It also became clear that the main effect of the patch antenna element is a limitation to the angles at which the antenna array can "beam". The preferred antenna element is a square patch antenna. The position of the feed point changes the ability of the array to beam in certain angles. An antenna array consisting out of differently orientated patches could be considered to enhance the ability to "beam" in more angles. An other alternative is a patch array where the feed point of the patch can be chosen.
Bibliography


